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THE MODAL STRUCTURE OF THE PRIOR-RESCHER FAMILY OF INFINITE PRODUCT SYSTEMS

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1. *Prior-Rescher Family of Product Systems.** Let S be an arbitrary sentential system of m-valued truth-functional logic, $m \geq 2$. Following the notational conventions of Rescher ([6], p. 99), we mean by $\Pi_k(S)$ the truth-functional system that is the k-fold product of S with itself. That is, the truth values of $\Pi_k(S)$ are the k-tuples of the truth values of S, and the semantics of $\Pi_k(S)$ is based on the semantics of S in the following way. Let $\otimes$ be an n-ary connective. Then $\otimes(\langle\alpha_1^1, \ldots, \alpha_k^1\rangle, \ldots, \langle\alpha_1^n, \ldots, \alpha_k^n\rangle)$ is $\langle\otimes(\alpha_1^1, \ldots, \alpha_1^n), \ldots, \otimes(\alpha_k^1, \ldots, \alpha_k^n)\rangle$. Rescher observes that there are two plausible ways to treat truth-value designation in $\Pi_k(S)$. One might regard a truth value $\langle\alpha_1, \ldots, \alpha_k\rangle$ as designated in $\Pi_k(S)$ iff (a) each member of $\langle\alpha_1, \ldots, \alpha_k\rangle$ is designated in S, or iff (b) at least one member of $\langle\alpha_1, \ldots, \alpha_k\rangle$ is designated in S. Both alternatives lead to exactly the same theses for all the product systems discussed in this paper, so it is a matter of indifference which is chosen. For the sake of definiteness we adopt alternative (a). Again following Rescher's notation (*ibid.*), by $\Pi_{\aleph_0}(S)$ we mean the denumerable product of S with itself. That is, the truth values of $\Pi_{\aleph_0}(S)$ are the denumerable sequences $(\alpha_1, \alpha_2, \alpha_3, \ldots)$ of the truth values of S, and the semantics of $\Pi_{\aleph_0}(S)$ is based on that of S in the same way that the semantics of $\Pi_k(S)$ is based on the semantics of S.

In [6], p. 195, Rescher considers the family of systems $\Pi_k(S)^+$ and $\Pi_{\aleph_0}(S)^+$, which we call the *Prior-Rescher family of product systems*. In all these systems the underlying truth-functional logic S has a "truest" designated value t and a "falsest" nondesignated value f. One obtains $\Pi_k(S)^+$ by supplementing $\Pi_k(S)$ with the singulary operator $\Box$ whose semantics is given as follows. The value of $\Box A$ is the k-tuple $\langle$t, ..., t$\rangle$ if the value of A is that same k-tuple; otherwise, the value of $\Box A$ is the k-tuple $\langle$f, ..., f$\rangle$. Similarly, one gets $\Pi_{\aleph_0}(S)^+$ by supplementing $\Pi_{\aleph_0}(S)$

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