Notre Dame Journal of Formal Logic
Volume XIII, Number 2, April 1972
NDJFAM

STRANGE ARGUMENTS

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In logic one frequently considers a set P of formal sentences together with a single formal sentence c and one asks whether c is a logical consequence of P. In teaching logic it is convenient to follow the philosophers (Mates [2]) and use the term *argument* to indicate such an ordered pair (P, c). If indeed c follows from P then (P, c) is said to be *valid* and otherwise *invalid*.

After learning the formal definition of the logical consequence relation in sentential logic (propositional calculus) students often find it "strange" that there should be valid arguments (P, c) whose *premises* P share no sentential letters with their respective *conclusions* c. Typical examples, of course, are the following facts:

(1) *q follows from* $\{p, \sim p\}$
(2) $(q \supset q)$ *follows from* $\{p\}$

Sometimes students are apt to attribute the "strangeness" to the concept of logical consequence and to feel, on the strength of the attribution, that the formal concept is incorrect, unrealistic, arbitrary, or something of the sort. It is the purpose of this note to indicate a nice way of disabusing thoughtful students of such unjustified feelings while at the same time providing them with some mathematical reasoning involving useful insight into the mathematical implications of the definition.

The Background Let D (the dictionary) be a countably infinite set of sentential letters and let L be the set of formal sentences built-up recursively from D using &, v, $\supset$ and $\sim$ as logical connectives and) and (for punctuation. As usual an *interpretation*, i, of L is function from D into the set $\{t, f\}$ of *truth-values* assigning a truth-value to each sentential letter. Given an interpretation i, truth-values *relative* to i (or *under* i or *on* i) are determined by defining a truth-valuation function V^i from L to $\{t, f\}$ as follows:

(1) $V^i x = ix$, *for each* x *in* D
(2) $V^i \sim x = NV^i x$

Received March 9, 1970