Notre Dame Journal of Formal Logic
Volume XIII, Number 2, April 1972
NDJFAM

THE PRAGMATICS OF FIRST ORDER LANGUAGES. I

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1. *Introduction: Pragmatic Interpretations* According to the well-known semiotic principle of Peirce, every signification is a triadic relation between *sign, object,* and *interpretant*. In this paper we shall investigate the structure of this relation, in the case where the signs are formulas of a first order language, their objects are taken to be elements in a model of the language, and their interpretants are taken to be certain valuing dispositions of the users of the language. We shall show that there are pragmatically definable linguistic models of the theory of polyadic Boolean algebras [1]. Then we may represent first order languages by such linguistic algebras, and their models by appropriate functional algebras; we shall represent the relevant valuing dispositions of the users of first order languages by means of pragmatic interpretations, which we now define.

Let $\mathbf{L}$ be the set of all finite sequences of the elements of a given non-empty finite set $\mathbf{L}_0$. Let $\mathbf{V}$ be a three-membered set, which we represent as $\{0, 1, 2\}$. Let $\mathbf{C}, \mathbf{U}, \mathbf{W}$ be arbitrary non-empty sets, disjoint from one another and from $\mathbf{L}$ and $\mathbf{V}$. Intuitively, $\mathbf{L}$ is the set of expressions of a language with alphabet $\mathbf{L}_0$, $\mathbf{C}$ is a set of conditions under which the expressions of $\mathbf{L}$ may be valued in the set $\mathbf{V}$ by the members of the set $\mathbf{U}$ of users of $\mathbf{L}$, and $\mathbf{W}$ is the set of times at which expressions of $\mathbf{L}$ may be valued. The elements 0, 1, 2 of $\mathbf{V}$ may be regarded as, respectively, the values *reject, accept, neither accept nor reject*. The set $\mathbf{C}$ may be regarded as the total evidence available to the users of $\mathbf{L}$, and subsets of $\mathbf{C}$ as partial evidence, for valuing the expressions of $\mathbf{L}$ in the set $\mathbf{V}$.

Let $\mathfrak{C}$ be the set of all subsets of $\mathbf{C}$. Let D be the set of all mappings d from the Cartesian product $\mathbf{U} \times \mathbf{W} \times \mathfrak{C}$ into $\mathbf{V}$ such that, for all $u, u' \in \mathbf{U}$; $w, w' \in \mathbf{W}$; $c \subset \mathbf{C}$:

(1.1) $d(u, w, c) = d(u', w', c)$ *if* $d(u, w, c) \neq 2 \neq d(u', w', c)$.

An element (u, w, c, v) of a mapping d in D may be regarded as a disposition of the user u, at time w, under the set of conditions c, to perform the valuation v. (1.1) is a condition of uniformity on the mappings d in D, in the

Received October 20, 1969