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AN ABBREVIATION OF CROISOT'S AXIOM-SYSTEM FOR DISTRIBUTIVE LATTICES WITH I

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In [2] there have been established the axiom-systems which satisfy certain formal requirements defined in that paper for distributive lattices with the constant elements. Unfortunately, only when [2] was already composed and in the final proofs, and, therefore, could not be changed, I unexpectedly obtained a rather interesting result which makes the deductions presented in [2] obsolete, although they are entirely correct. Namely, I have proved that in the sets of postulates given in the assumptions of Theorem 2, *cf.* [2], section **3**, axiom *A17* is redundant.

1 It is obvious, that if an algebraic system

$$\mathfrak{S} = \langle A, \cap, \cup, I \rangle$$

with two binary operations $\cap$ and $\cup$, and with a constant element $I \epsilon A$, is a distributive lattice with I, then the following formulas

S1 $[a] : a \epsilon A . \supset . I = a \cup I$ [i.e. *A1* in [2], section **2**]
S2 $[a] : a \epsilon A . \supset . a = a \cap I$ [i.e. *A2* in [2], section **2**]
S3 $[abc] : a, b, c \epsilon A . \supset . a \cap ((b \cap b) \cup c) = (c \cap a) \cup (b \cap a)$ [i.e. *A4* in [2], section **2**]

are provable in the field of $\mathfrak{S}$. I shall prove here the converse of this statement. Namely:

If the system $\mathfrak{S}$ satisfies the formulas S1, S2 and S3, then it is a distributive lattice with I.

Proof: Let us assume *S1*, *S2* and *S3*. Then:

S4 $[ab] : a, b \epsilon A . \supset . a = (I \cap a) \cup (b \cap a)$ [*S2*, *S1*, *S3*; as *A5* in [2], section **2**]
S5 $[abc] : a, b, c \epsilon A . \supset . (b \cap c) \cup (a \cap c) = c \cap ((b \cup a) \cup (b \cup a))$ [*S3*, *S4*, *S3*, *S2*; as *A6* in [2], section **2**]
S6 $[a] : a \epsilon A . \supset . I \cap (a \cup a) = a$ [*S4*, *S5*, *S2*, *S4*; as *A7* in [2], section **2**]
S7 $[a] : a \epsilon A . \supset . I \cup a = I$ [*S2*, *S3*, *S1*; as *A8* in [2], section **2**]

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