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A PROPER SUBSYSTEM OF S4.04

BOLESŁAW SOBOCIŃSKI

It is self-evident that, in the field of modal system S4.04 which has been established in [8]*, the following formula

Ł1 $\mathfrak{C}\mathfrak{C}\mathfrak{C}pLppCLMLpp$

is easily provable. It will be proved in this note:

1. that the addition of **Ł1**, as a new axiom, to S4 generates a system, called S4.02, which is a proper extension of S4 and at the same time is properly contained in each of the systems S4.04 and S4.1,
2. that S4.02 neither contains the systems S4.2, S4.3 and S4.3.2 nor is contained in any one of them,
3. and that the addition of **Ł1**, as a new axiom, to each of the systems K1 and Z1 generates the systems which are inferentially equivalent to K1.1 and Z3 respectively.

Proof:

1 Each of the matrices $\mathfrak{M}5$, $\mathfrak{M}7$, $\mathfrak{M}9$ and $\mathfrak{M}11$ which are given in [3], p. 350, verifies S4, but:

(i) $\mathfrak{M}5$ verifies **Ł1**, but falsifies **G1**, *cf.* [2], section **4.2.** Hence, $\mathfrak{M}5$ also falsifies **D1** and **F1**.

(ii) $\mathfrak{M}7$ verifies **F1** and **K1**, but falsifies **Ł1** for $p/3$: $= \mathfrak{C}\mathfrak{C}\mathfrak{C}3L33CLML33 = \mathfrak{C}\mathfrak{C}LC343CLM43 = \mathfrak{C}\mathfrak{C}L23CL13 = \mathfrak{C}LC43C13 = \mathfrak{C}L13 = LC13 = L3 = 4$.

(iii) $\mathfrak{M}9$ verifies **Ł1**, but falsifies **L1**, *cf.* [2], section **4.4.**

(iv) $\mathfrak{M}11$ verifies **Ł1**, but falsifies **N1**, *cf.* [5], p. 383.

2 It follows immediately from the considerations which are given in section **1** that:

*An acquaintance with the papers which are cited in this note and, especially, with the enumeration of the extensions of S4 and their proper axioms given in [3], pp. 247-350, in [4], and in [2], is presupposed.

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