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A NEW CLASS OF MODAL SYSTEMS

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In [15][1], p. 354, Zeman considered a formula

Z1 $\mathfrak{C}LMpCLMqCMKpqLMKpq$ [In [15] formula (52)]

which, obviously, is a consequence of S5, and at the same time is verified by Lewis-Langford Group II. The addition of **Z1**, as a new axiom, to the systems S4.3.1, S4.3.2 and S4.4 generates the modal systems which Zeman calls S4.3.3, S4.3.4 and S4.6 respectively. And, as it has been proved by Zeman, the systems S4.3.3 and S4.3.4 are distinct, and each of them is a proper subsystem of S4.6 which in its turn is contained in his system S4.9, i.e., in Schumm's system S4.7, *cf.*, [5], [15], [8] and [9]. Hence, S4.6 is a consequence of my system V1 and, at the same time and independently, of S5. In [15] Zeman has remarked that formula **Z1** is clearly provable in the field of K3.1 or of K3.2, since each of those systems contains the formula

K2* $\mathfrak{C}LMpCLMqLMKpq$ [In [15], p. 354, formula (54)]

which according to Zeman, *cf.* [15], p. 349, formula (15), in the field of S4.4, is inferentially equivalent to

K1 $\mathfrak{C}LMpMLp$.

But, it is self-evident that in the field of S4 formula **K2*** is inferentially equivalent to McKinsey's formula, *cf.* [1], p. 92, formula (F),

K2 $\mathfrak{C}LMpCLMqMKpq$

which, as I have proved in [13], pp. 77-78, section **5**, in the field of S4 is inferentially equivalent to **K1**. Therefore, any system belonging to the family $\mathcal{K}$ of non-Lewis modal systems, *cf.* a definition of this family in [10], p. 313, contains formula **Z1**. Whence, besides Zeman's systems S4.3.3, S4.3.4 and S4.6 which are mentioned above, there can be other

1. An acquaintance with the papers which are cited in this note and, especially, with, *cf.* [8], pp. 347-350, the enumeration of the extensions of S4 and their proper axioms, is presupposed.

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