

A STUDY OF SOME SYSTEMS IN THE NEIGHBORHOOD OF S4.4.

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This paper investigates S4.4 and some closely related systems both semantically and proof-theoretically. In what follows, we:

- (1) Set down a characteristic matrix for S4.4.
- (2) Extend the methods employed in (1) above to show that "Group II" of Lewis and Langford [3] is characteristic for the system K4 of Sobociński [9].
- (3) Set down and investigate semantically and proof-theoretically a system S4.9 which is between S4.4 and S5; there is no system properly contained in S5 and properly containing S4.9.

In [10], Sobociński introduced the system S4.4, which is $S4 + \mathcal{C}pCMLpLp$; S4.4 properly includes Prior's Diodorean system D ($S4.3 + \mathcal{C}\mathcal{C}\mathcal{C}pLppCMLpp$); this is shown in [10], where D is called S4.3.1 (for a thorough discussion of D, see [6, p. 20 ff.]). D is a modal system whose modal operators may be considered defined in a time framework. It is the logic in which "necessarily α " means " α is true now and will be at every instant in the future," and "possibly α " means " α either now is true, or will be at some instant in the future." [6] discusses also time interpretations for several systems included in D; S4.3, S4.2, and S4; also considered is a time interpretation for S5. The time sequence for S4.3 is linear and connected as is that of D, but S4.3 differs from D in having its time sequence continuous while that for D is composed of discrete instants. The sequences for S4.2 and S4 are not connected, but permit "branching" (some statements which are possible now can turn out never to be true). S4.2 differs from S4 in requiring "convergence" of its branchings—all statements which are "possibly necessary" eventually become necessary. While these systems define their modal operators in terms of the future alone, "necessarily α " for S5 is defined as " α is now true, and always will be, and always was," and "possibly α " is "either α now is true, or at some instant in the future it will be true, or at some instant in the past it was true."

Since S4.4 is "surrounded" by systems with time interpretations, one may wonder if it is possible to come up with such an interpretation of it. I