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# MATRIX SATISFIABILITY AND AXIOMATIZATION

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The appearance of *Polish Logic 1920-1939* (edited by Storrs McCall) is an event of considerable importance for logicians interested in the development of modern symbolic logic.[1] In conjunction with Tarski's *Logic, Semantics, and Metamathematics,* this collection of papers makes the central early source material from the Polish school of logicians available in English translation.[2] There are, however, a few matters of fit between the volumes which have escaped scrutiny. This is in no way intended to be a criticism of McCall's editorial decisions. Within the limits of a single volume of source papers, his choices seem uniformly excellent. In this paper I would like to discuss one theorem which is stated in [3] without proof, and no proof for which occurs in the papers which were chosen for inclusion in [2]. This theorem seems worthy of discussion because of the interesting connection which it establishes between matrix characterizations of propositional calculi and equivalent axiomatic systems.

In their paper "Investigations into the Sentential Calculus," J. Łukasiewicz and A. Tarski state the following theorem about the arbitrary calculus $L_n$ $(2 \leqslant n < \aleph_0)$:[3]

*Let* $\mathfrak{M} = \langle A, B, f, g\rangle$ *be a normal matrix in which the set* $A \cup B$ *is finite. If the sentences 'CCpqCCqrCpr', 'CCqrCCpqCpr', 'CCqrCpp', 'CCpqCNqNp', 'CNqCCpqNp' are satisfied by this matrix, then the set of sentences satisfying* $\mathfrak{M}$ *may be finitely axiomatized.*

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1. See [2].
2. See [3].
3. See [1], p. 50. A normal matrix in which $B$ is $\{1\}$ defines the calculus $L_n$ when the number of values $n$ is identical with the number of values $A$ in the matrix. Strictly, a normal matrix could have more than one designated value, so that Wajsberg's theorem applies to a larger class of calculi than the calculi $L_n$. As only the calculi $L_n$ have assumed an important role in the literature, we will ignore this complication in what follows except for one remark preceding Lemma 10.

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