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MEASURABLE CARDINALS AND CONSTRUCTIBILITY WITHOUT REGULARITY

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It has been shown (see Dana Scott [5]) that the axiom of constructibility ($V = L$) is incompatible with the existence of a measurable cardinal number. In [4] we gave a decomposition of $V = L$, over set theory without the axiom of regularity, into the axiom of regularity and the proposition:

$$\mathsf{P}\colon\ \forall x(x \in V \wedge x \subset L \rightarrow x \in L).$$

In this paper we will show that even without the axiom of regularity **P** is sufficient to insure that there are no measurable cardinals. We shall work within the system of [1] but use the notation of [5]. Our result is thus formulated as follows:

Theorem I. *In* **GB** *set theory with* **AC** *but without the axiom of regularity, if* **P** *holds, then there does not exist a measurable cardinal.*

Our proof will follow that of Scott [5], who assumed $V = L$ in the following form:

(*) *If M is a class such that*

(i) $M \subset \mathcal{P}(M) \subset \bigcup_{x \in M} \mathcal{P}(x)$

(ii) $x - y, \bigcup x, \breve{x}, x|y, E|x \in M$, *for all* $x, y \in M$;

then $V = M$.

(In the above, $\mathcal{P}$ denotes the power set operation so $\mathcal{P}(M)$ is the class of all subsets of M; $\bigcup x = \bigcup_{y \in x} y$; $\breve{x}$ denotes the operation of forming the converse of the relational part of x; $x|y$ denotes the operation of forming the relative product of the relational parts of x and y; $E|y = \{\langle u, v\rangle\colon u \in v \in x\}$.)

We shall formulate **P** in a similar way. We first note that a set x is called *grounded* if there does not exist an infinite descending $\in$-chain beginning with x.

Proposition II. *In the field of* **GB** *set theory with* **AC** *but without the axiom of regularity the following statements are equivalent:*

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