

## WEAK FORMS OF THE AXIOM OF CONSTRUCTIBILITY

RICHARD L. POSS

### TABLE OF CONTENTS

Introduction . . . . .	257
§1. Constructibility . . . . .	257
§2. General results . . . . .	258
§3. Notation and conventions . . . . .	259
§4. Forcing. . . . .	259
Chapter I. Constructible sets of integers . . . . .	261
Chapter II. Constructible subsets of ordinals. . . . .	273
Chapter III. Constructibility without regularity . . . . .	289
Bibliography. . . . .	298

### INTRODUCTION\*

§1. *Constructibility.* The notion of constructibility in set theory was first mentioned by Kurt Gödel in [7], in the year 1938. Roughly speaking, he said that a set is "constructible" if it can be obtained from the empty set by the elementary set operations applied transfinitely many times. We then say that if every set in our theory is constructible the axiom of constructibility holds. Gödel used the axiom of constructibility to prove the consistency of the axiom of choice and of the generalized continuum hypothesis (under the assumption that set theory itself is consistent). To do this, he exhibited a model in which the axiom of constructibility holds and then showed that the axiom of constructibility implies the generalized continuum hypothesis and, hence, the axiom of choice.

---

\*This work is based on a dissertation submitted in partial fulfillment of the requirements for the Ph.D. degree in Mathematics at the University of Notre Dame, August, 1970. The author wishes to express his gratitude to Prof. Bolesław Sobociński for directing this research.