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A TABLEAU PROOF METHOD ADMITTING THE EMPTY DOMAIN

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1 *Introduction.* There have been several papers concerning systems of first order logic whose theorems are valid in all domains including the empty one. Some, for example, [1, 2] do not admit vacuous quantification. If vacuous quantification is allowed, two definitions of validity in the empty domain are possible, depending on how vacuous quantification is interpreted. Mostowski [5] interprets $(\forall x)A$, where x does not occur free in A, as equivalent to A; Hailperin [3] and Quine [6] interpret $(\forall x)A$ as true over the empty domain. All the preceeding proof systems are axiomatic, however see [4] for a natural deduction system.

In this paper we present simple and intuitive modifications of the tableau proof system of [8] (allowing vacuous quantification): one which produces a logic equivalent to that of Hailperin and Quine, and one which produces a logic equivalent to Mostowski's. We first sketch the classical system, then we present our modifications and sketch proofs of correctness and completeness.

2 *The Classical Tableau System.* We use $x, y, z, \ldots$ for individual variables (free and bound); $a, b, c, \ldots$ for individual parameters; and $A, B, C, \ldots$ to represent formulas. The notion of formula is defined as usual, allowing vacuous quantification. By $A(x/a)$ we mean the result of substituting the parameter a for every free occurrence of the variable x in A. A formula with no free variables is called a closed formula, or a sentence. A formula with no parameters is called pure.

We use the unified notation of Smullyan [7, 8] in which α stands for any essentially conjunctive formula, β for any disjunctive, γ for any universal formula, and δ for any existential. In the charts below we list the four α forms, and give the respective components, denoted α_1 and α_2, and the three β forms and their respective components, denoted β_1 and β_2.

α	α_1	α_2
$A \wedge B$	A	B
$\sim(A \vee B)$	$\sim A$	$\sim B$
$\sim(A \supset B)$	A	$\sim B$
$\sim\sim A$	A	A

β	β_1	β_2
$A \vee B$	A	B
$\sim(A \wedge B)$	$\sim A$	$\sim B$
$A \supset B$	$\sim A$	B

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