Notre Dame Journal of Formal Logic
Volume XII, Number 2, April 1971

INDEPENDENCE OF THE AXIOMS AND RULES OF INFERENCE OF ONE SYSTEM OF THE EXTENDED PROPOSITIONAL CALCULUS

NADEJDA GEORGIEVA

In [1] A. Church introduced an extended propositional calculas **P**, built up by a logical operator, $\supset$ (implication), an universal quantifier and propositional variables. The only operator variables in **P** are propositional variables.

The axioms of **P** are the three following:

A1. $p \supset q \supset . q \supset r \supset . p \supset r$
A2. $p \supset q \supset p \supset p$
A3. $p \supset . q \supset p$

The primitive rules of inference are:

R1. $\dfrac{A \supset B, A}{B}$ (modus ponens)

R2: $\dfrac{A}{\check{S}_B^p A \,\Big|}$. (rule of substitution)

R3. $\dfrac{A \supset B}{A \supset (a)B}$

R4. $\dfrac{A \supset (a)B}{A \supset B}$

In R3 and R4 a is a propositional variable, which is not free in A.

The purpose of this work is to show, that the axioms and rules of **P** are independent.

1. *Theorems* Now we go on to the proof of some theorems of **P**.

1. $\vdash p \supset p$

By A1, R2, A3 and R1:
$\vdash q \supset p \supset r \supset . p \supset r$
$\vdash p \supset q \supset p \supset p \supset . p \supset p$

Hence by A2 and R1:
$\vdash p \supset p$

2. $\vdash p \supset [p \supset q] \supset . p \supset q$

By A1, R2 and R1:

Received February 25, 1970