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TWO MODES OF DEDUCTIVE INFERENCE

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1. This study is a sequel to the author's [1], which tried to exhibit a system of natural deduction as a mere typographical variant of an axiom system G. The aim was to provide a proof technique combining the formal advantages of deduction from axioms with the intuitive advantages of deduction from assumptions or premises.

To some readers, however, the central notion of a *context*—and the notation embodying it—were psychologically opaque or unmanageable in practice. Furthermore, the conventions bridging the axiomatic method and the method of natural deduction were jerry-built, piecemeal and by example. Thus, the paper failed to meet some reasonable standards of simplicity and directness. For these reasons, a fresh approach seems in order.

The burden of the sequel, therefore, is to rejustify the claim that the best features of the axiomatic method and of the method of natural deduction may be secured within the framework of the first alone; by formulating a new axiom system G′ that is tractable to generalized deductive routine, by showing that some straightforward conventions for rewriting its formulae yield a method of proof indistinguishable in practice from well known techniques of natural deduction, and by proving that the system is both complete and sound in the sense that all and only valid quantificational formulae are among its theorems.

G′ and its metalanguage are entirely new. The choice of primitives is in line with popular tastes, the notion of context is abandoned in favor of a more transparent descriptive device (2.1) and the major link between the axiomatic and natural methods of deduction is forged in one stroke by recursion (6.1). The presentation is so ordered as to facilitate comparison with that of the parental essay; nevertheless, it is entirely self-contained, thus sparing those with no interest in comparative anatomy the labor of repeated cross-reference.

2. The primitives of G′ are 0-place predicate letters (sentence letters) 'p', 'q', 'r', 's' and their subscripted variants, m-place predicate letters ($m \geq 1$) 'F^m', 'G^m', 'H^m' and their subscripted variants, variables 'w', 'x', 'y', 'z' and their subscripted variants, the negation sign '-', parentheses, the conditional sign '$\supset$' and the universal quantifier sign '$\forall$'.

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