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A NOTE ON THE INTUITIONIST FAN THEOREM

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The purpose of this note is to point out that the Intuitionist Fan Theorem as stated in the literature mentioned is classically false. A classical counterexample to the Theorem is given. It is pointed out that the modified Fan Theorem does not give rise to a classical contradiction mentioned in Heyting [5].

The usual statement of the Fan Theorem* is, following [5],

If S is an Intuitionist fan and φ an integer valued function defined for every element δ of S then a natural number N can be computed for $\langle S, \varphi\rangle$ such that for any element δ of S, $\varphi(\delta)$ is determined by the first N components of δ.

We refer to this as the weak theorem. The counter example we presently introduce leads us to modify the above statement to the strong theorem,

If S is an Intuitionist fan and φ an integer valued function defined for every element δ of S such that $\varphi(\delta)$ is determined by a finite number of components of δ, then a natural number N can be computed such that for any element δ of S, $\varphi(\delta)$ is determined by the first N components of δ.

Consider the fan S whose elements are infinitely proceeding sequences (ips) $\{d_n\}$. The spread law SL is as follows:

(i) 0 *is an admissible* 1-*sequence*,

(ii) 0, 1 *are admissible* n-*components for any* $n \geq 2$, *that is, given an admissible* $(n-1)$-*sequence* $d_1, \dots, d_{n-1}$, *we may choose* $d_n = 0$ *or* $d_n = 1$ *subject to*

(iii) *if* $d_{n-1} = 1$ *then* $d_n = 1$ *for any* $n \geq 2$.

The complementary law CL, assigns to any admissible n-sequence d_1, $d_2, \dots, d_n$, the number d_n. Clearly S is a fan and may be represented by the following tree:

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*See: [1] p. 430; [2], p. 462; [3], p. 143; [4], p. 15; and [5], p. 42.

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