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THE DEPENDENCE OF A MEREOLOGICAL AXIOM

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In this note we show that in the standard axiom system for mereology which follows, the reflexive axiom, *M2*, is dependent on *M3*, *DM*, *M4*, and *M5*.

M1. $[AB]: A \varepsilon \mathbf{el}(B) . \supset . B \varepsilon B.$

M2. $[A]: A \varepsilon A . \supset . A \varepsilon \mathbf{el}(A).$

M3. $[ABC]: A \varepsilon \mathbf{el}(B) . B \varepsilon \mathbf{el}(C) . \supset . A \varepsilon \mathbf{el}(C).$

DM. $[Aa] \therefore A \varepsilon \mathbf{Kl}(a) . \equiv : A \varepsilon A : [D]: D \varepsilon a . \supset . D \varepsilon \mathbf{el}(A) : [D]:$
$D \varepsilon \mathbf{el}(A) . \supset . [\exists EF] . E \varepsilon a . F \varepsilon \mathbf{el}(D) . F \varepsilon \mathbf{el}(E).$

M4. $[Aa]: A \varepsilon a . \supset . [\exists B] . B \varepsilon \mathbf{Kl}(a).$

M5. $[ABa]: A \varepsilon \mathbf{Kl}(a) . B \varepsilon \mathbf{Kl}(a) . \supset . A \varepsilon B.$

P1. $[AD]: A \varepsilon A . D \varepsilon \mathbf{el}(A) . \supset . [\exists F] . F \varepsilon \mathbf{el}(D)$

PF $[AD]$: Hyp(2) $. \supset .$
$[\exists B] .$
3) $B \varepsilon \mathbf{Kl}(A) .$ [*M4*, 1]
4) $A \varepsilon \mathbf{el}(B) .$ [*DM*, 3, 1]
5) $D \varepsilon \mathbf{el}(B) .$ [*M3*, 2, 4]
$[\exists F] . F \varepsilon \mathbf{el}(D)$ [*DM*, 3, 5]

P2. $[A]: A \varepsilon A . \supset . A \varepsilon \mathbf{Kl}(\mathbf{el}(A))$ [*DM*, $a/\mathbf{el}(A)$, E/D, *P1*]

P3. $[ABD]: A \varepsilon A . B \varepsilon \mathbf{Kl}(A) . D \varepsilon \mathbf{el}(B) . \supset . [\exists EF] . E \varepsilon \mathbf{el}(A) .$
$F \varepsilon \mathbf{el}(D) . F \varepsilon \mathbf{el}(E)$

PF $[ABD]$: Hyp(3) $. \supset .$
4) $A \varepsilon \mathbf{el}(B) .$ [*DM*, 2, 1]
$[\exists EF] .$
5) $E \varepsilon A .$
6) $F \varepsilon \mathbf{el}(D) .$ [*DM*, 2, 3]
7) $F \varepsilon \mathbf{el}(E) .$
8) $E = A .$ [5, 1]
9) $F \varepsilon \mathbf{el}(A) .$ [7, 8]
10) $F \varepsilon \mathbf{el}(B) .$ [*M3*, 6, 3]
$[\exists G] .$
11) $G \varepsilon \mathbf{el}(F) .$ [*DM*, 2, 10]
12) $G \varepsilon \mathbf{el}(D) .$ [*M3*, 11, 6]
$[\exists EF] . E \varepsilon \mathbf{el}(A) . F \varepsilon \mathbf{el}(D) . F \varepsilon \mathbf{el}(E)$ [9, 12, 11]

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