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REMARKS ON THE W. C. NEMITZ'S PAPER "SEMI-BOOLEAN LATTICES"

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In his paper [9] W. C. Nemitz considers the semi-Boolean lattices. We will show in this note that the classes of all semi-Boolean lattices and of all relative Stone lattices with a greatest element are equal. At the end we shall consider some equalities concerning the Brouwerian lattices.

We will start with some preliminaries. A Brouwerian (or implicative) lattice is a lattice L in which, for any given elements a and b, the set of all $x \in L$ such that $a \cap x \leq b$ contains a greatest element $a * b$, the relative pseudo-complement of a in b. It is known (see [2]) that any Brouwerian lattice is distributive and contains a greatest element 1. A lattice L with a least element 0 is called *pseudo-complemented* if for every element $x \in L$ there exists a relative pseudo-complement $x * 0$ which is denoted by x^{**}. A pseudo-complemented lattice need not be distributive but it always contains a greatest element 1. A *Stone lattice* is a distributive pseudo-complemented lattice which satisfies the equality

(1) $$x^* \cup x^{**} = 1 \textit{ for all } x \in L.$$

A lattice is called *relative Stone* if all its (closed) intervals are Stone lattices.

In [1] or, more generally, in [5] the following statements were proved

Lemma 1. *A lattice L is a Brouwerian lattice if and only if the following conditions are satisfied.*

(i) *L is a distributive lattice with* 1;
(ii) *All (closed) intervals of L are pseudo-complemented.*

Lemma 2. *A lattice L with* 1 *is a relative Stone lattice if and only if it is a Brouwerian lattice which satisfies the following equality*

(2) $$(x * y) \cup (y * x) = 1 \textit{ for all } x, y \in L.$$

A Brouwerian lattice L is called semi-Boolean (see [9]) if for all $x, y \in L$

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