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THE COMPLETENESS OF COPI'S SYSTEM OF NATURAL DEDUCTION

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I. *Introduction.* This note will outline a proof of the completeness of the system of sentential logic developed by Copi in [2] which also provides an effective proof-method for this system. Although the completeness of the Copi system is well known, the method to be used here does not involve a detour through an auxiliary axiomatic system (as in [1], where the completeness of the system presented in [3] is established). Since the method is of some interest in itself, the general procedure is sketched first.

Let $P_1, P_2, \ldots, P_n, Q$ be any sequence of sentential schemata. Then a sentential system of natural deduction is here said to be *complete* if and only if there is a derivation in the system of Q from $P_1, P_2, \ldots, P_n$ whenever the schema $(P_1 \cdot P_2 \cdot, \ldots, \cdot P_n) \supset Q$ is a (standard) truth-table tautology. The notion of a derivation used here will, of course, depend on the particular rules of inference or rules of replacement which are peculiar to the system under study. In the method used below, completeness is demonstrated as follows. First, we show that any tautology is derivable in the system from any non-empty sequence of sentences whatsoever. It now follows as a corollary that $(P_1 \cdot P_2 \cdot, \ldots, \cdot P_n) \supset Q$ is derivable from $P_1, P_2, \ldots, P_n$ whenever $(P_1 \cdot P_2 \cdot, \ldots, \cdot P_n) \supset Q$ is a tautology. Repeated use of the rule of conjunction (or an equivalent device) will now yield $(P_1 \cdot P_2, \ldots, \cdot P_n)$. A single application of *modus ponens* (i.e., the rule of detachment) then gives us Q, the desired result. In what follows, it is assumed that the reader is familiar with the inference and replacement rules of [2], here called **CND** (Copi's system of natural deduction), along with their abbreviations.[1]

1. The system of natural deduction developed in [3] is called **CMD** by Canty in [1]. The method of proving the completeness of **CND** developed here is not immediately applicable to **CMD**, however, due to the fact that the rule of Absorption (*Abs.*) is dropped in that system and replaced by rules of Conditional Proof (**C.P.**) and Indirect Proof (**I.P.**). These last rules are so formulated as to prevent the *continuation* of the proof once they are applied, and this is crucial to the method presented here.

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