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NON-EXISTENCE DOES NOT EXIST

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The main aims of this paper are to explain criteria for the identity of individuals, to compare various criteria for the existence of properties and for the existence of propositions, and to present certain theses concerning the existence and identity of individuals, of propositions, and of properties. Several other topics are, however, treated incidentally; for example an extended sentential logic designed to take care of certain semantical paradoxes and truth-value gaps by allowing for statement-incapable sentences is sketched.

In order to attack in a formal way the question of the existence of properties and relations and to formalise widely employed criteria for the existence of attributes, i.e. of properties and relations, an extended predicate calculus must first be introduced. As a first move it is valuable to determine how much can be done in the simplest and most accessible of higher order functional calculi, viz. second-order functional calculus. Now this logic has to be so designed that it can express such propositions as "Some properties do not exist" and "All properties, whether possible or impossible, (e.g. exist)". At first this suggests that a system like \mathbb{R}^* , which allows for quantification over all possible individual items, be extended to second order.¹ Such an extension of \mathbb{R}^* to second order can be obtained by

(i) relaxing a formation rule of \mathbb{R}^* so that predicate and propositional variables as well as individual variables may be bound, i.e. by replacing the formation rule specifying how ' π ' (read 'for all possible') can enter into wff by the rule:

if A is a wff then $(\pi u)A$ is a wff, where u is any variable (individual, predicate, propositional).

(ii) replacing 'individual variable' whenever it occurs in the axioms and transformation rules of R^* by 'variable' and using in these axioms and rules extra-systematic (or syntactic) variables which range over both individual, predicate or propositional variables. These extensions are, however, insufficient to yield a Henkin-complete second-order predicate logic.

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