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THE PRODUCT OF IMPLICATION AND COUNTER-IMPLICATION SYSTEMS

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1. *Introduction.** Rasiowa has obtained in [1] a finite axiomatization of the product system of "implication" and "equivalence". In this paper, we show that the logic system based on the single binary connective ○ with the logical matrix

○	1	2	3	4
*1	1	1	3	3
2	2	1	4	3
3	1	1	1	1
4	2	1	2	1

that is the product connective of implication (C) and counter-implication (Ɔ) is finitely axiomatizable. The axiom and the rules of inference have been obtained by combining the axioms and the rules of inference of the complete axiomatizations of the implication system (C-system) and of the counter-implication system (Ɔ-system).

2. *Preliminary definitions.* In these definitions Δ and Δ_1 are arbitrary binary connectives.

2.1. *Δ-formulas.* Δ-formulas are defined recursively as follows:

i) a sentential variable, a small Roman letter, is a Δ-formula;

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