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NEW ALGORITHMS FOR THE STATEMENT AND CLASS CALCULI

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INTRODUCTION.

Efficient algorithms for making inferences in the statement calculus and in the calculus of classes may be developed with a fractional representation of statements. The idea is to take the transitivity of implication as the fundamental mode of inference. We seek then a representation in which hypothetical syllogism, which is an expression of this transitivity, appears in a convenient form. Both the inferences:

$$\begin{array}{ll} \textit{All } A \textit{ are } B & p \supset q \\ \textit{All } B \textit{ are } C & q \supset r \\ \textit{All } A \textit{ are } C & p \supset r \end{array}$$

work as if one "cancelled" the middle terms. This apparently simple-minded view can, as we shall show, be developed into a remarkably simple and perspicuous set of techniques for manipulation of logical relations between calsses and between statements.[1]

PART I: THE STATEMENT CALCULUS.

1. *Fractional Representation of the Statement Connectives.* With a few stipulated conventions of interpretation some of the basic logical relationships between the connectives of the statement calculus may be built into the notation itself. We represent the usual statement connectives in the following "fractional" form[2]:

$$p : \frac{1}{p} \qquad \sim p \ : \frac{1}{p'}$$

$$p \supset q : \frac{p}{q} \qquad p \vee q \ : \frac{1}{pq}$$

$$p \equiv q : \frac{p}{q} \cdot \frac{q}{p} \qquad p \cdot q : \frac{1}{(p \cdot q)}$$

It is convenient to adopt some arithmetical terminology in talking about

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