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AFFINE GEOMETRY WITH S. DOWDY'S "TRAPEZOID" AS PRIMITIVE

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In [1] S. Dowdy introduces an axiom system for affine geometry based on the primitive $\mathbf{t}(ABCD)$ which intuitively means that A, B, C, D are the vertices of a trapezoid. In this paper the system is first simplified and then altered slightly so that the defined terms which appear in the axioms can be eliminated and still produce a "reasonable" looking system. A system in which $\mathbf{c}(ABC)$, A, B, C are collinear, is the only relation which appears is then given.

The system $\mathbf{T}$ which appears in this paper is Dowdy's $\mathbf{A}*$ in [1]. System $\mathbf{T}'$ is derived from $\mathbf{T}$ by the following simplifications: Two disjuncts are removed from *D2*, one conjunct is removed from the last disjunct of *D3*, *A3* is eliminated, the equivalence in *A4* is replaced by an implication, *A5* is replaced by a shorter simpler axiom, and a conjunct is removed from the antecedent of *A8*. System $\mathbf{T}''$ is obtained from $\mathbf{T}'$ by shortening and at the same time strengthening the definition of collinearity so that *A5'*, the transitivity of collinearity, follows from *A6*, the transitivity of parallelism. The theses prefixed with an *L* are to be found in [1], pp. 245-255.

1. SYSTEM T

D1 $[A] : A\varepsilon\alpha . \equiv . [\exists BCD] . \mathbf{t}(ABCD)$

D2 $[AB] .\therefore \mathbf{r}(AB) . \equiv : [\exists CD] : \mathbf{t}(ABCD) . \mathrm{v} . \mathbf{t}(ACBD) . \mathrm{v} . \mathbf{t}(CBAD)$

D3 $[ABC] .\therefore \mathbf{c}(ABC) . \equiv : \mathbf{r}(BC) : A = B . \mathrm{v} . A = C . \mathrm{v} . [\exists XY] . \mathbf{t}(BCXY) . \mathbf{t}(BAXY) . \mathbf{t}(CAXY)$

A1 $[\exists ABCD] . \mathbf{t}(ABCD)$

A2 $[ABCD] : \mathbf{t}(ABCD) . \supset . A \neq B$

A3a $[ABCD] : \mathbf{t}(ABCD) . \supset . \mathbf{t}(DCAB)$

A3b $[ABCD] : \mathbf{t}(ABCD) . \supset . \mathbf{t}(ABDC)$

A4 $[ABC] :: A\varepsilon\alpha . B\varepsilon\alpha . C\varepsilon\alpha . \supset .\therefore \sim\mathbf{c}(CAB) : \equiv : [\exists D] . \mathbf{t}(ABCD) . \mathrm{v} . A = B$

A5 $[ABCMN] : A \neq B . \mathbf{c}(AMN) . \mathbf{c}(BMN) . \mathbf{c}(CMN) . \supset . \mathbf{c}(CAB)$

A6 $[ABCDEFG] : \mathbf{t}(ABCD) . \mathbf{t}(ABEF) . \mathbf{t}(CDEG) . \supset . \mathbf{t}(CDEF)$

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