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E AND S4

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In [5], Anderson and Belnap use their system E *of Entailment*, enriched with sentential quantifiers, to develop modal and non-modal theories of *enthymematic implication* which coincide in part with theories based on the Lewis calculus S4 and the Heyting calculus **HJ** respectively. The present essay will show that the results relating E and S4 can be sharpened considerably. Specifically, I shall show that strict implication can be exactly defined in E in two different senses, and that these correspond to a strong and a weak form of S4-deducibility. A sequel will show that relations analogous to those established here hold between the non-modal analogue R of E and the systems **HJ**, **HD**, and **HK**.[1]

I take all theories to be formulated in a common language L, with '$\rightarrow$', '&', '$\forall$', and '$-$' primitive, along with an unspecified *finite* number of sentential variables $p_1, \ldots, p_n$.[2] A, B, C, etc., shall be arbitrary formulas, built up as usual from sentential variables and primitive connectives. For all A and B, $A = B$ shall serve as a definitional abbreviation of $(A \rightarrow B)$ & $(B \rightarrow A)$; in symbols, $A = B =_{\mathrm{df}} (A \rightarrow B) \& (B \rightarrow A)$. I rank the connectives, including those defined below, thus in order of increasing scope: '&', '$\vee$', '$\dashv$', '$\supset$', '$\rightarrow$', '$\equiv$', '$=$', with '&' often omitted. Dots are sometimes used as in [11]. Otherwise omitted parentheses are restored by association to the left.

A formula A is in *apodictic form* iff (a) A is of the form $B \rightarrow C$, or (b) A is of the form B & C, where both B and C are in apodictic form. I sometimes write $A\square$ if A is in apodictic form.

I choose the following axioms for E.[3]

E1.	$A \rightarrow A$	Identity
E2.	$(A \rightarrow B) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow C))$	Transitivity
E3.	$(A \rightarrow (A \rightarrow B)) \rightarrow (A \rightarrow B)$	Contraction
E4.	$(A \rightarrow (B\square \rightarrow C)) \rightarrow (B\square \rightarrow (A \rightarrow C))$	Restricted Permutation
E5.	$AB \rightarrow A$	&E
E6.	$AB \rightarrow B$	&E
E7.	$(A \rightarrow B)(A \rightarrow C) \rightarrow (A \rightarrow BC)$	&I

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