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A CHARACTERIZATION OF A SPHERICAL m-ARRANGEMENT

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In [1] a simplified definition of an open m-arrangement was presented. The purpose of this paper is to present a simpler characterization of a spherical m-arrangement than that presented in [2], a characterization which because of its similarity to the characterization of an open m-arrangement in [1] leads us to define a new type of structure, an (n,m)-arrangement, of which open m-arrangements and spherical m-arrangements are but special cases. The principal result to be proved in this paper in the following:

Theorem 1: *Let X be a topological space with geometry G of length $m - 1 \geq 0$. Then X and G form a spherical m-arrangement if and only if the following conditions are satisfied:*

i) *Each* 0*-flat consists of precisely two points.*
ii) *If f is a k-*1*-flat and g is a k-flat with $f \subseteq g$, then f disconnects g into two non-empty convex components which are open in g, $0 \leq k \leq m$.*
iii) *Each* 1*-flat is connected.*
iv) (If f is an m-1-flat, then we call the components of X-f *half-spaces* of X.) *The collection of half-spaces of X forms a subbasis for the topology of X.*

Proof: We note first that i) and ii) are the same as 1) and 5) in the definition of a spherical m-arrangement given in [2]. We now show that i) through iv) also imply 2), 3), and 4) in the definition of a spherical m-arrangement. In the following propositions then we assume that we have a space X with geometry G of length m-1 which satisfies i) through iv).

Proposition 1: *X is* T_1.

Proof: Each m-1-flat is closed and any 0-flat $\{x,y\}$ is the intersection of finitely many m-1-flats, and hence is closed. But by ii) $\{x,y\}$ is disconnected; hence it follows that $\{x\}$ and $\{y\}$ are both closed sets. Since any one point subset of X is contained in some 0-flat, X is T_1.

Proposition 2: *If f is any 1-flat and x is a point of f, then x is a non-cut point of f.*

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