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AN EXTENSION OF NEGATIONLESS LOGIC

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§1. Nelson [1] has provided a formalization of part of Griss' negationless mathematics [2]. The logic Nelson devised uses a quantified implication $(A \supset \overline{x}\ B)$ and a quantified disjunction $(\Sigma\overline{x}(A_1, \ldots, A_n))$ as well as &, $\forall$, and $\exists$. These connectives do not exhaust the possibilities for rendering each provable sequent of Nelson's $\mathbf{P}_1$ system as a provable formula: when given a sequent $A_1, \ldots, A_m \rightarrow B_1, \ldots, B_n$, we lack a corresponding closed formula to be read negationlessly as "*for all* $x_1, \ldots, x_k$ *if* A_1 *and* . . . *and* A_m, *then* B_1 *or* . . . *or* B_n." Further, in Nelson's two most restricted predicate calculi there is no obvious way of forming Griss negation in several variables. If $\neq$ is a distinguishability relation and $P(t_1, \ldots, t_n)$ is a formula in which $x_1, \ldots, x_n$ do not occur, then the Griss negation of $P(t_1, \ldots, t_n)$ should be read "*for all* $x_1, \ldots, x_n$ *if* $P(x_1, \ldots, x_n)$ *then* $x_1 \neq t_1$ *or* . . . *or* $x_n \neq t_n$."

We have defined a general connective which provides the lacking notation [3]. Using the notation of [1] we give the definition and introduction rules for this connective. Let $\overline{x}$ be a non-empty list of distinct variables, Ψ a (possible empty) list of formulas, and Φ a non-empty list of formulas: then $(\Psi \supset\overline{x}\ \Phi)$ is a formula. Introduction rules suitable to P_2 - A_2 are

$$\frac{\Gamma, \Pi(\overline{x}) \rightarrow \Psi(\overline{x}) \mid \Gamma \rightarrow (\exists\overline{z})(\Pi(\overline{z})_1 \& \ldots \& \ \Pi(\overline{z})_m) \mid \Gamma \rightarrow (\exists\overline{z})\Psi(\overline{z})_1 \mid \ldots \mid \Gamma \rightarrow (\exists\overline{z})\Psi(\overline{z})_n}{\Gamma \rightarrow (\Pi(\overline{z}) \supset\overline{z}\ \Psi(\overline{z}))}$$

and

$$\frac{\Sigma \rightarrow A_1(\overline{t}), \Lambda \mid \ldots \mid \Sigma \rightarrow A_m(\overline{t}), \Lambda \mid B_1(\overline{t}), \Omega \rightarrow \Phi \mid \ldots \mid B_n(\overline{t}), \Omega \rightarrow \Phi}{(A_1(\overline{x}), \ \ldots, \ A_m(\overline{x}) \supset\overline{x}\ B_1(\overline{x}), \ \ldots, \ B_n(\overline{x})), \Sigma, \Omega \rightarrow \Lambda, \ \Phi}$$

in which Γ does not contain any of $\overline{x}$ free, each variable of $\overline{x}$ (term of $\overline{t}$) is free for the corresponding variable of $\overline{z}$ ($\overline{x}$) in each formula of $\Pi(\overline{z}), \Psi(\overline{z})$ $(A_1(\overline{x}), \ldots, B_n(\overline{x}))$, if $\Pi(\overline{x})$ $(A_1(\overline{t}), \ldots, A_m(\overline{t}))$ is empty then the premise(s) not involving $\Psi(\overline{x})$ $(B_1(\overline{t}), \ldots, B_n(\overline{t}))$ is (are) omitted, $\Pi(\overline{x})$ is a list of m formulas, $\Psi(\overline{x})$ is a non-empty list of n formulas, etc. An additional premise

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