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EQUATIONAL CHARACTERIZATION OF NELSON ALGEBRA

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1. *INTRODUCTION.* H. Rasiowa in [8] and [9] has introduced the notion of **N**-lattice which plays a rôle in the study of the constructive logics with strong negation considered by David Nelson [7] and A Markov [4]. Not all axioms used by H. Rasiowa to characterize **N**-lattices, here called Nelson algebras, are equations. A paper published in collaboration with A. Monteiro, [3], gives a characterization of these algebras by equations but the proofs are heavily based on results indicated in [6] which have been obtained using transfinite induction. The purpose of this work, done under the guidance of Dr. A. Monteiro, is to indicate a purely arithmetical proof of that result. We reproduce here known results with the object of making this paper self-contained.

2. *THE DEFINITION OF H. RASIOWA.* Let us consider, in first place, the following definition;

2.1. *DEFINITION. A system $\langle A, 1, \sim, \wedge, \vee\rangle$ constituted by 1°) a non empty set A, 2°) an element $1 \in A$ 3°) a unary operator $\sim$ defined on A, 4°) two binary operations, $\wedge$ and $\vee$, defined on A, will be called a quasi-boolean algebra,* [1], *or a Morgan algebra,* [5], *if the following conditions are verified:*

N1. $x \vee 1 = 1$
N2. $x \wedge (x \vee y) = x$
N3. $x \wedge (y \vee z) = (z \wedge x) \vee (y \wedge x)$
N4. $\sim\sim x = x$
N5. $(x \wedge y) = \sim x \vee \sim y$

A system $\langle A, \wedge, \vee\rangle$ verifying axioms N2 and N3 is, according to M. Scholander [10], a distributive lattice, from N1 we deduce that 1 is the last element of A. We can prove:

N'2. $a \vee (a \wedge b) = a$
N'3. $a \vee (b \wedge c) = (c \vee a) \wedge (c \vee b)$
N'5. $\sim(a \vee b) = \sim a \wedge \sim b$

and that $0 = \sim 1$ is the first element of A.

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