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ON A MODAL SYSTEM OF D. C. MAKINSON AND B. SOBOCIŃSKI

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It is well-known that if N, K and M are taken as primitive, with C and L defined in the usual manner, the theses of Prior's Diodorean system D = S4.3 + $CLCLCpLppCMLpp$[1] can be characterized as those, and only those, formulas verified by the matrix $\mathfrak{P} = < V, d, -, \cap, P >$, where:

1. V is the set of all ω sequences $(x_0, x_1, \ldots)$ of 0's and 1's.
2. d is the designated element $(1, 1, 1, \ldots)$ of V.
3. $-$ and $\cap$ are operations in V defined in pointwise fashion from the familiar Boolean operations $-$ and $\cap$ in $\{0, 1\}$.
4. P is the operation in V such that if $(x_0, x_1, \ldots) \varepsilon V$, then $P(x_0, x_1, \ldots) = (y_0, y_1, \ldots)$ where, for each i, $y_i = 1$ iff $x_j = 1$ for some $j \geq i$.

In [2], Makinson observes that if 4. is replaced by

5. P^* is the operation in V such that if $(x_0, x_1, \ldots) \varepsilon V$, then $P^*(x_0, x_1, \ldots) = (y_0, y_1, \ldots)$ where, for each i, $y_i = 1$ iff $x_j = 1$ for some $j \leq i$.

then D*, defined as the system for which the resulting matrix $\mathfrak{P}^* = < V, d, -, \cap, P^* >$ is characteristic, is a proper extension of D and, like D, admits of a very natural tense-logical interpretation.

We here show that D* can be axiomatized and is equivalent to the system K3.1 = S4.3 + $CLCLCpLppp$ discussed by Sobociński in [4]. To this end, let S = D + $CLMpMLp$. It is readily established that S $\subseteq$ K3.1 $\subseteq$ D*—use the known fact that $CLMpMLp$ is a thesis of K3.1 and note that $CLCLCpLppp$ and $CpCMLpp$ yield $CLCLCpLppCMLpp$—and so it will suffice to show that D* $\subseteq$ S.

Suppose $\gamma_1, \ldots, \gamma_m$ are the subformulas of α. Then for each γ_i, we put $\beta_i = MKC\gamma_iL\gamma_iCN\gamma_iLN\gamma_i$ and let β be the conjunction of all β_i's. Where μ is any assignment into $\mathfrak{P}$ or $\mathfrak{P}^*$ and $\mu(\delta) = (x_0, x_1, \ldots)$, we let $\mu_j(\delta) = x_j$.

Lemma 1. *If* $\vdash_{\mathrm{D}} C\beta\alpha$, *then* $\vdash_{\mathrm{S}} \alpha$.

Proof. Using the matrix $\mathfrak{P}$ it is easily checked that $CCLMpMLpMKCpLpCNpLNp$ is a thesis of D and therefore of S. Then since $\vdash_{\mathrm{S}} CLMpMLp$, we

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