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EXISTENCE AND IDENTITY IN QUANTIFIED MODAL LOGICS

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§1. The aim of this paper is to present a way in which philosophical objections to the development of a combined quantification and modal logic based on **S5** can be overcome. In more detail, the objectives are to show that **S5** is immune to criticisms directed at those theorems which distinguish it from **S4** and **T**; that problematic theorems[1] of modalised predicate logic like the Barcan formulae $[\Diamond(\exists x)f(x) \supset (\exists x)\Diamond f(x)]$ and $[\Diamond(\exists t)g(x, t) \supset (\exists t)\Diamond g(x, t)]$ can be appropriately qualified once existence is explicitly treated; that puzzles over identity can be escaped by a more elaborate treatment of identity than the standard treatment; and that difficulties associated with quantification into modal sentence contexts can be cleared away given these treatments of existence and identity. A combination of these moves suffice, so it will be argued, to meet standard objections, most forcefully presented by Quine[2], to quantified modal logics. Admittedly a full elaboration of these moves calls for some sentence/statement distinction, some analytic/synthetic (or necessary/contingent) distinction, and some sense/designation (or connotation/denotation) distinction: but although, consequently, it is not to be expected that a combination of these moves will satisfy Quine, they may satisfy some who have been disturbed by the objections Quine raises.

§2. *Existence in a first-order modalised predicate logic.* A semantical system **S5R*** is obtained by adjoining modal postulates for **S5** (with primitive symbol '$\Box$') to a system **R*** of first-order predicate logic (with primitive quantifier 'Π'). **R*** differs from usual quantification theory in having the predicate constant 'E', read 'exist(s)', added to its primitive symbols, and in interpretation: in place of the frequent interpretation - $[(\Pi x)f(x)]$ is true if f is true of all existent items of the domain selected - the following interpretation - $[(\Pi x)f(x)]$ is true if f is true of all possible (consistently describable or designable) items of the domain selected - is preferred[3]. The postulate set of **R*** is as follows:

R0. *If A is truth-functionally valid, then A is a theorem.*

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