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A SUBSTITUTION FREE AXIOM SET FOR SECOND ORDER LOGIC

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In what follows we present an adequate formulation of second order logic by means of an axiom set whose characterization does not require the notion of proper substitution either of a term for an individual variable or of a formula for a predicate variable. The axiom set is adequate in the sense of being equivalent to standard formulations of second order logic, e.g., that of Church [1]. It is clear and need not be shown here that every theorem of the present formulation is a theorem of the formulation given by Church. It of course will be shown here, however, that each of Church's axioms are theorems of the present system and that each of his primitive inference rules is either a primitive (and only *modus ponens* is taken as a primitive rule here) or a derived rule of the present system.

The importance of obtaining an axiomatic formulation such as herein described lies partly in the significance of reducing any axiom set to an equivalent one which involves fewer metalogical notions, especially such a one as proper substitution. However, of somewhat greater importance, it is highly desirable that we possess a formulation of both first and second order logic which can be extended without qualification to such areas as tense, epistemic, deontic, modal and logics of the like. Now proper substitution especially has been the main obstacle to such unqualified extensions of standard logic, and we take it to be of no little significance that at least for first order logic (with identity) a substitution free axiomatic formulation has been provided.[1] The present system extends this earlier result to the level of second order logic.[2]

A second difficulty in unqualified extensions of standard logic concerns the form which Leibniz' law, i.e., the law regarding interchangeability *salva veritate*, is to take. Generally, in the extensions of standard logic to modal logic, this law has been formulated in an unqualified form applicable to all contexts, thereby lending credence to the questionable view that only "intensions" or the like can serve adequately as values of the variables for such systems. In the substitution free formulations of first order logic

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