

NOTE ON THE C-CALCULUS

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In this note we remark that the Tarski-Bernays and Frege systems which are obtained by H. Hiž in his C-calculus presentation [1] can be obtained in fewer theorems. In the beginning we consider the first 15 theses, the same ones that are in [1], then the derivation is made in the following way:

- 11 $p/Cpq, q/Cqr, r/Cpr \times C13-16$
16. $CCqrCCpqCpr$
 $13 p/Csq, q/CCqrCsr, r/CCpCqrCpCsr \times C13 p/s-C16 q/Cqr,$
 $r/Csr-17$
17. $CCsqCCpCqrCpCsr$
 $11 p/Csq, q/CpCqr, r/CpCqr \times C17-18$
18. $CCpCqrCCsqCpCsr$
 $13 p/Crs, q/CCqrCqs, r/CCpCqrCpCqs \times C16 p/q, q/r,$
 $r/s-C16 q/Cqr, r/Cqs-19$
19. $CCrsCCpCqrCpCqs$
 $19 p/CpCqr, q/Cpq, r/CpCpr, s/Cpr \times C8 q/r-C18 s/p-20$
20. $CCpCqrCCpqCpr$

4, 15, 13 form the Tarski-Bernays system and 4, 20 form the Frege system (in [1] 4, 33). We remark also that the deduction theorem can be proved without using thesis 20 (see [2]), the one that permitted this thesis to be obtained in the C-calculus.

REFERENCES

- [1] Hiž, H., "A completeness proof for C-calculus," *Notre Dame Journal of Formal Logic*, vol. XIV (1973), pp. 253-258.
- [2] Milici, C., "A remark on axiom-system for the classical two-valued $\{\rightarrow, \neg\}$ propositional logic," *Glasnik Matematički*, vol. 9 (1974), pp. 3-5.

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