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A NOTE ON REFLEXIVENESS

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The job of working through the algebra of relations is not made any easier by the fact that authors frequently use key terms differently. This is to be regretted; however, the important thing is that you be able to keep these usages straight in your own mind. What follows then is an attempt to get straight on one particular aspect of the algebra of relations—the notion of reflexiveness.

The most studied properties of relations are relexivity, symmetry, and transitivity. Variances appear from the very start; for whereas all authors define "symmetric" and "asymmetric" as:

$$\mathsf{Sym}\ R =_{df} (x)(y)(xRy \rightarrow yRx)$$
$$\mathsf{Asym}\ R =_{df} (x)(y)(xRy \rightarrow \sim yRx),$$

some authors (Carnap, e.g.) define "nonsymmetry" simply as "$\sim \mathsf{Sym}\ R$", and others (Copi, e.g.) define it "$\sim \mathsf{Sym}\ R$ & $\sim \mathsf{Asym}\ R$". And there is an analogous variance in the transitivity triad. For whereas all authors define "transitive" and "intransitive" as:

$$\mathsf{Tra}\ R =_{df} (x)(y)(z)\ [(xRy\ \&\ yRz) \rightarrow xRz]$$
$$\mathsf{Intra}\ R =_{df} (x)(y)(z)\ [(xRy\ \&\ yRz) \rightarrow \sim xRz],$$

some authors (again Carnap) define "nontransitive" simply as "$\sim \mathsf{Tra}\ R$", and others (again Copi) define it "$\sim \mathsf{Tra}\ R$ & $\sim \mathsf{Intra}\ R$". The reasons for adopting one version of nonsymmetry (or nontransitivity) instead of the other are most likely pragmatic, but we shall let this pass.

The variance already noted, of course, has an analogy in the reflexivity group. Some authors define "nonreflexive" simply as "$\sim \mathsf{Refl}\ R$", while others define it "$\sim \mathsf{Refl}\ R$ & $\sim \mathsf{Irrefl}\ R$". Yet the map is smudged even more by variant definitions of "$\mathsf{Refl}\ R$". Here are the most frequently used definitions of reflexiveness: (giving just the definiens)

(A) $(x)xRx$

(B) $(x)(y)[xRy \rightarrow (xRx\ \&\ yRy)]$

(C) $(x)[(\exists y)(xRy \vee yRx) \rightarrow xRx]\ .$

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