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ANOTHER SYSTEM OF NATURAL DEDUCTION

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In the pages* that follow a system of natural deduction is described and shown to be adequate. Among the noteworthy features of the system are the perfect symmetry and intuitive plausibility of the restrictions that govern applications of the rules **UG** and **EI**. These features are made possible through the use of a precisely defined notion of arbitrariness. With one exception deductions run no longer than those of other commonly taught systems. The exception is the system found in the second edition of Quine [3].[1] But, it is perhaps to be expected that somewhat longer deductions are the price that must be paid to avoid Quine's devices of flagging and ordering.

We assume a system of sentences (well-formed formulas having no free occurrences of variables) built up in familiar ways from predicate and name letters together with apparatus for truth functions, existential quantification, and universal quantification. A *deduction* is to be understood as any finite sequence of ordered couples generated by rules that will shortly follow. But first, here are some needed definitions. Where $\langle \mu, A \rangle$ is the k'th term of a deduction $\mathfrak{D}$, $\langle \mu, A \rangle$ will be referred to as the *k'th line of* $\mathfrak{D}$, members of μ will be referred to as *premise numbers of* the k'th line of $\mathfrak{D}$, and A will be said to *occur as* or to be *written as* the k'th line of $\mathfrak{D}$. Where j is a premise number of line k, the sentence occurring as the j'th line of $\mathfrak{D}$ will be said to be a *premise of* the k'th line of $\mathfrak{D}$. And, finally, a name letter will be said to be *arbitrary for* the k'th line of $\mathfrak{D}$ if it occurs neither in any premise of that line nor in any earlier line obtained by **EI**.

In what follows 'n' and 'm' are restricted to name letters, $n/m\ B$ is the result of replacing each occurrence of m in B by an occurrence of n, and

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1. A variation of Quine's system can be found in Massey [1].

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