Notre Dame Journal of Formal Logic
Volume XVI, Number 3, July 1975
NDJFAM

CONCERNING THE POSTULATE-SYSTEMS OF SUBTRACTIVE ABELIAN GROUPS

BOLESŁAW SOBOCIŃSKI

It is rather well-known, *cf.*, e.g., [5], p. 256, that in the field of Groups Theory, instead of operation $a + b$, the inverse operation $a - b$, which is defined as follows: $a - b = c$.≡. $a = b + c$ for all elements a, b, and c of the given algebra, can be accepted as a sole primitive notion for Abelian Groups. Throughout this paper the Abelian Groups based on this inverse operation will be called the Subtractive Abelian Groups. In [5] A. Tarski has established two postulate systems for such algebras. And, recently, in [1], R. Güting constructed another axiomatization for these systems. In [2] and [3] S. Leśniewski accepted a ternary functor $\varphi(a\,b\,c)$ defined as follows: $\varphi(a\,b\,c)$.≡. $a + b = c$ for all elements a, b, and c of the given algebra, as a sole primitive functor and has used it to construct two single axioms for Groups Theory and Abelian Groups Theory, respectively. Analogously, a ternary functor defined as follows: $\boldsymbol{\rho}(a\,b\,c)$.≡. $a - b = c$ for all elements a, b, and c of the given algebra, can be accepted as a sole primitive notion for Subtractive Abelian Groups. And, in [5], p. 256, Tarski announced without a proof that formulas *T1* and *T2*, presented in section 1 below, can be accepted as an axiom-system for such algebras.

A main purpose of this paper is to present a proof that formula *S1*, see section 1, which I constructed in the style of Leśniewski's axioms mentioned above, can be accepted as a single axiom of the Subtractive Abelian Groups. As a by-product of the deductions given below it will also be shown that all postulate-systems mentioned above are inferentially equivalent.[1]

1 Although it is not necessary, in order to present the deductions which follow in the next sections in a uniform way, we accept here that the carrier sets which occur in two formalizations $\mathfrak{G}_1$ and $\mathfrak{G}_2$ of Subtractive Abelian Groups represent the same unempty arbitrary set.

1. This paper is written in the style of Leśniewski's works [2] and [3].

Received August 15, 1970
Final version received March 12, 1975