Notre Dame Journal of Formal Logic
Volume XVI, Number 3, July 1975
NDJFAM

COMPACTNESS IN ABSTRACTIONS OF POST ALGEBRAS

R. BEAZER

Introduction An algebra $\langle A; F\rangle$ is said to be *equationally compact* if the existence of a simultaneous solution of any finite subset of any set Σ of polynomial equations (with constants) in A implies the existence of a simultaneous solution of Σ. An algebra $\langle A; F\rangle$ is said to be *topologically compact* if A is endowed with a compact, Hausdorff topology under which all the operations in F are continuous. The problem of determining the equationally compact algebras and the answer to Mycielski's question (see [15]), "Is every equationally compact algebra a retract of a topologically compact algebra?" for a particular class of algebras, is usually a difficult one. The problem has been solved for semilattices in [11] and [3], for Boolean algebras in [17], and for Post and Post-like algebras in [2]. In recent years, several abstractions of Post algebras have been studied. The purpose of this note is the characterization of the equationally compact algebras in some of these classes.

Preliminaries A *Brouwerian algebra* is an algebra $\langle A; \vee, \wedge, \rightarrow\rangle$, where $\langle A; \vee, \wedge\rangle$ is a lattice and $\rightarrow$ is a binary operation such that $x \wedge y \leq z$ *if and only if* $x \leq y \rightarrow z$. Every Brouwerian algebra is distributive and has a greatest element 1. A *Heyting algebra* is a Brouwerian algebra with a least element 0. In a Heyting algebra A, the element $x \rightarrow 0$ will be denoted by x^* and is the *pseudocomplement* of x in A. The set $\mathsf{S}(A) = \{x \in A;\ x = x^{**}\}$ forms a Boolean algebra; *the algebra of closed elements* of A. The set $\mathsf{D}(A) = \{x \in A;\ x^* = 0\}$ forms a filter; *the filter of dense elements* in A. A bounded, distributive, pseudocomplemented lattice satisfying the identity $x^* \vee x^{**} = 1$ is called a *Stone algebra*. In any Stone algebra A, $\mathsf{S}(A)$ coincides with the centre $\mathsf{C}(A)$ of A. An L-*algebra* is a Heyting algebra satisfying the identity $(x \rightarrow y) \vee (y \rightarrow x) = 1$. Any L-algebra is a Stone algebra and satisfies the identity $x \vee y = ((x \rightarrow y) \rightarrow y) \wedge ((y \rightarrow x) \rightarrow x)$. Consequently, the operation $\vee$ can be omitted from the set of fundamental operations. For the connection between L-algebras and logic, the reader is referred to [13].

Received August 29, 1974