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S1 ≠ S0.9

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In both [1] and [3] it was conjectured that S0.9 is weaker than S1, but there was no proof that this is so. In what follows we see that this is so using Hintikka's model set model system semantics (see [2]).

Consider the systems defined in terms of the following axiom schemata and rules as in [4].

A1: $A \supset (B \supset A)$
A2: $(A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))$
A3: $(\sim A \supset \sim B) \supset (B \supset A)$
A4: $\Box A \supset A$
A5: $\Box(A \supset B) \supset (\Box(B \supset C) \supset \Box(A \supset C))$
A6: $\Box(A \supset B) \supset (\Box A \supset \Box B)$

R1: $\dfrac{A, A \supset B}{B}$ R2: $\dfrac{\Box(A \supset B) \ \& \ \Box(B \supset A)}{\Box(\Box A \supset \Box B) \ \& \ \Box(\Box B \supset \Box A)}$ R3: $\dfrac{\Box(A \supset B)}{\Box(\Box A \supset \Box B)}$

We use the standard definitions of $\Diamond$, &, v, and $\equiv$, and we use $\Box$Ai($1 \leqslant i \leqslant 6$) for schema resulting from schema Ai by prefixing the symbol $\Box$ before the whole of Ai in brackets.

We define four modal systems:

S0.5 = {A4, A6, □A1 - □A3; R1}
S0.9 = {A4, □A1 - □A4, □A6; R1, R2}
S1 = {A4, □A1 - □A5; R1, R2}
S2 = {A4, □A1 - □A4, □A6; R1, R3}

It has been shown that S0.5 is included in S0.9, and S0.9 is included in S1, and both S0.9 and S1 are included in S2 (see [3]).

We now construct a Hintikka type model $\langle \Omega, C_S \rangle$ where Ω is a model system of model sets $\Omega = \{\mu_1, \mu_2, \ldots, \mu_n, \ldots\}$ ($n \geqslant 1$), and where C_S is a set of consistency conditions, for some system S, for deciding which formulae of the system S can be included (or imbedded) in any μ_n. The membership of C_S is drawn from:

1. If μ_n contains an atomic formula it does not contain its negation.
2. If $(A \supset B) \in \mu_n$ then $\sim A \in \mu_n$ or $B \in \mu_n$ or both.

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