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THE GENERAL DECISION PROBLEM FOR MARKOV ALGORITHMS WITH AXIOM

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*Introduction** Let $\mathcal{M}_{\mathcal{A}}$ denote the general decision problem for Markov algorithms with axiom. Of interest to us is whether or not this class of problems is as richly structured, with regard to degrees of unsolvability, as those classes studied in Hughes, Overbeek, and Singletary [2]. In this paper we shall present proofs which show this to be so. In particular we shall show that the general decision problem for the range of total recursive functions is many-one reducible to $\mathcal{M}_{\mathcal{A}}$ and consequently that every r.e. many-one degree of unsolvability is represented by $\mathcal{M}_{\mathcal{A}}$. Furthermore we shall show this result to be best possible, with regard to degree representation, in that every r.e. one-one degree is not represented by this family of decision problems. And finally we shall demonstrate a simple application of these results to the study of splinters.

Preliminaries A semi-Thue system S is a pair $\langle \Sigma, P \rangle$ where Σ is a finite alphabet and P is a finite set of rules each of which is of the form $\alpha \rightarrow \beta$, for α and β words over Σ. For any arbitrary pair of words W_1, W_2 over Σ, we say that W_2 is an immediate successor of W_1 in S, denoted $(W_1, W_2)_S$, if there exist a pair of words U, V over Σ and a rule $\alpha \rightarrow \beta$ in P such that $W_1 \equiv U\alpha V$ and $W_2 \equiv U\beta V$. W_2 is said to be derivable from W_1 in S, denoted $W_1 \vdash_S W_2$, if either

(i) $W_1 \equiv W_2$,

or

(ii) there exists a finite sequence $V_1, \ldots, V_k$, where $k > 1$, of words over Σ such that $W_1 \equiv V_1$, $W_2 \equiv V_k$, and $(V_i, V_{i+1})_S$, for $i = 1, \ldots, k - 1$.

A Markov alogorithm M is a pair $\langle \Sigma, P \rangle$ where Σ is a finite alphabet and $P = \{\alpha_i R_i \beta_i | 1 \leqslant i \leqslant m\}$ is a finite ordered set of rules where

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