Notre Dame Journal of Formal Logic
Volume XVI, Number 1, January 1975
NDJFAM

TWO IDENTITIES FOR LATTICES, DISTRIBUTIVE LATTICES AND MODULAR LATTICES WITH A CONSTANT

SABURO TAMURA

In his paper [3] J. A. Kalman has defined lattices using two identities and six variables. We shall define lattices using two identities and five variables in Theorem 1. In Theorem 2 we shall give an axiom system for lattices with 0 consisting of two identities. J. Sholander's axiom system for distributive lattices with 0 contains three identities (*cf.*, [5]), but our axiom system in Theorem 3 consists of two identities. In Theorem 4 we shall give a definition for distributive lattices with 1 in the Croisot-Sobociński style (*cf.*, [1] and [7]). Finally, as axiom system for modular lattices with 0 shall be given in Theorem 5. In the remarks, axiom systems for lattices, distributive lattices and modular lattices with two constants are given by three identities.

Theorem 1. *Any algebraic system* $\langle A; \cdot; +\rangle$ *with two binary operations* $\cdot$ *and* $+$, *which satisfies the following two identities*

L1. $a = ba + a$
L2. $((ab)c + d) + e = ((bc)a + e) + (b + d)d$

is a lattice

Proof: We can prove it as Kalman has shown in [3] (*cf.*, Theorem 2 in this paper).

Theorem 2. *Any algebraic system* $\langle A; \cdot; +, 0\rangle$ *with two binary operations* $\cdot$ *and* $+$, *and with a constant* 0, *which satisfies the following two identities*

L1. $a = ba + a$
L2'. $(((0 + a)b)c + d) + e = ((bc)a + e) + (b + d)d$

is a lattice with 0.

Proof:
3. $c + a = (((0 + a)b)c + c) + a = ((bc)a + a) + (b + c)c = a + (b + c)c$ [L1, L2', L1]
4. $c + a = a + (bc + c)c = a + cc$ [3, L1]

Received May 24, 1974