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UNARY PREDICATES

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The following concerns what seems to be a mistake in the construction of a formal semantics for the predicate calculus. I have found this mistake in three books: Mendelson's [3], Shoenfeld's [4], and Leblanc and Wisdom's [1]. Other books verge on the mistake; of course, I cannot claim to have searched all developments of formal semantics. These three books differ in metalogical terminology. I shall follow the usage of Mendelson; but for purposes of cross-reference when a term is introduced, I shall indicate in parentheses the terms used by the other authors. When definitions differ among the authors only in terminology, I shall quote only Mendelson; but I shall provide in the footnote page references for all three books.

Any formal semantics involves two steps: An *interpretation* ([4]: *structure*; [1]: *D - interpretation*) assigns elements from a particular non-empty set, the *domain* ([4]: *universe*), to certain elements of the syntax including *individual constants* ([4]: *constants* (i.e., *O - ary functions*); [1]: *terms*) and *predicate letters* ([4]: *predicate symbols*; [1]: *predicates*). Predicate letters have associated with them a certain positive integer [Shoenfeld also permits O] which is the *degree* of the predicate letter; a predicate letter of degree n is an *n-ary* or *n-place* predicate. The second step is a definition of *satisfaction* ([4]: *truth*; [1]: *truth on a D - interpretation*) in terms of an interpretation.

In their respective definitions of satisfaction all three systems treat atomic *wfs* ([4]: *closed formulas*; [1]: *statements*) constructed from unary predicates as a special case of atomic wfs constructed from n-ary predicates:

If $\mathcal{A}$ is an atomic wf $A_j^n(t_1, \ldots, t_n)$ and B_j^n is the corresponding relation ([4]: predicate; [1]: subset of n-tuples on the domain) of the interpretation, then the sequence s satisfies $\mathcal{A}$ if and only if $B_j^n(s^*(t_1), \ldots, s^*(t_n))$, i.e., if the n-tuple $(s^*(t_1), \ldots, s^*(t_n))$ is in the relation B_j^n.[1]

1. [3], p. 51; [4], p. 19; [1], p. 307.

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