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ON THE THEORY OF INCONSISTENT FORMAL SYSTEMS

NEWTON C. A. da COSTA

Introduction This is an expository work,* in which we shall treat some questions related to the theory of inconsistent formal systems. The exposition will be neither rigorous nor complete. For details, the reader may consult the works cited in the References. (With reference to the historical aspects of the theory, see specially [1].) In general, the terminology, the notations, etc., are those of Kleene's book [17], with evident adaptations.

A formal system (deductive system, deductive theory, . . .) **S** is said to be inconsistent if there is a formula A of **S** such that A and its negation, $\neg A$, are both theorems of this system. In the opposite case, **S** is called consistent. A deductive system **S** is said to be trivial if all its formulas are theorems. If there is at least one unprovable formula in **S**, it is called non-trivial.

If the underlying logic of a system **S** is the classical logic (the intuitionistic logic, . . .), then **S** is trivial if, and only if, it is inconsistent. Hence, employing such a category of logics, the inconsistent systems do not present any proper logico-mathematical interest. Usually, we try to change the inconsistent theories to transform them into consistent ones. It is clear that under this transformation, some characteristic properties of a given inconsistent theory must be preserved; for instance, the common formal systems of set theory preserve certain traits of inconsistent naive set theory.

Nonetheless, there are certain cases in which we might think of studying directly an inconsistent theory. For example, a set theory containing Russell's class (the class of all classes which are not members of themselves) as an existing set, or a theory whose aim be the systematization of Meinong's theory of objects.[1] Apparently, it would be as

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1. Meinong's theory is discussed, for example, by Russell (*cf.* [21] and the articles by Meinong, Ameseder and Mally cited there). One of the objections formulated by Russell against Meinong's theory is precisely that it implies a derogation of the principle of contradiction.

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