Notre Dame Journal of Formal Logic
Volume XV, Number 3, July 1974
NDJFAM

A SIMPLE PROOF OF HERBRAND'S THEOREM

ANDRÉS R. RAGGIO

Hilbert and Bernays (*cf.*, *Grundlagen der Mathematik*, vol. II) prove Herbrand's theorem using their first ε-theorem. We can avoid this step by employing, instead of a Hilbert-type formalization of logic, the Beth's tableaux. But as we cannot dispense with function signs, we must supplement their usual rules in three cases. a) Existential quantifier to the right: in the next line of the tableau we must write down all substitutions of the corresponding bound variable by terms built up out of all free variables and function signs which have already occurred in the tableau. We get in the general case an effectively denumerable list of formulae; the strict finitistic character of Beth's tableaux is lost but the procedure is thoroughly constructive. b) Universal quantifier to the left: the corresponding change. c) Cancellation of identical members of a disjunction.

The proof of completeness of the semi-formal Beth's tableaux follows the well known pattern.

Given a formula of quantificational logic in prenex normal form

$$\begin{array}{cc} \text{prefix} & \text{nucleus} \\ \end{array}$$

$$(1) \qquad \wedge_x \wedge_y \vee_z \wedge_h \vee_m \wedge_l \mathfrak{A}(x, y, z, h, m, l)$$

According to Hilbert and Bernays' proof, we must substitute in the nucleus the variables which are bound by universal quantifiers occurring at the beginning of the prefix by different free variables not occurring in (1). All other variables of the nucleus which are bound by universal quantifiers must be substituted by different function signs which have as many arguments as there are existential quantifiers in the prefix preceding the corresponding universal quantifier. The arguments of these function signs must be filled by the bound variables of the preceding existential quantifiers. In this way we get

$$(2) \qquad \vee_z \vee_m \mathfrak{A}(a_1, a_2, z, \phi(z), m, \psi(z, m))$$

Let us suppose that (1) has a closed development (*cf.*, P. Lorenzen: "Dialogkalküle," *Archiv für mathematische Logik und Grundlagenforschung*, Bd. 15/3-4, (1972)), then (2) has also a closed development. In its n'th line

Received December 3, 1973