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α-MODELS AND THE SYSTEMS T AND T*

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This paper[1] is the fourth (and last) of a series in which we study two systems of set theory, T and T*, which were designed to serve as foundations for category theory (*cf.* [3], [4], and [5]). It is divided into two parts; in the first, we develop and make more precise certain results of section 3 of [5]. As the subject matter of the first part is of independent interest, it is treated here in some detail; nonetheless, the full development of the subject will appear elsewhere. In the second part, we only outline how T and T* can be used as foundations for the notions of category and functor, since it is not difficult to work out the details.

I. α-Models

1 Introduction Let us consider a first-order language $\mathcal{L}$ containing the family $(\iota)_{i<\alpha}$ of (distinct) constant terms, where α is an ordinal (in the sense of von Neumann) greater than 0, and a set Γ of sentences of $\mathcal{L}$ (in particular, Γ can be a first-order theory). An α-model of Γ is a model in the ordinary sense, such that every element of it is denoted by at least one term ι_i, $i \in \alpha$. A sentence F of $\mathcal{L}$ is said to be a semantic α-consequence of Γ if it is true in every α-model of Γ. Then, it seems natural to ask the following question: Is it possible to strengthen the first-order predicate calculus, with or without equality, in such a manner as to assure that if F is a semantic α-consequence of Γ, then F is also a syntactic consequence of Γ in the new strengthened calculus? A strengthened version $\mathcal{C}$ of the predicate calculus is called α-complete if, and only if, whenever a sentence F is a semantic α-consequence of a set Γ of sentences, we also have that F is a syntactic consequence of Γ in $\mathcal{C}$.

The concept of α-model appears when we consider theories in which it is natural to suppose that their intuitive models must satisfy the condition

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