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A RESULT OF EXTENDING BOCHVAR'S 3-VALUED LOGIC

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In this note I shall adopt the notation that Nicholas Rescher uses in [1]. Thus lower case Roman letters are meta-variables, and lower case Greek letters are object variables. We begin with Bochvar's basic system B_3:

p	$\neg p$
T	F
I	I
F	T

	$p \wedge q$			$p \vee q$			$p \rightarrow q$			$p \leftrightarrow q$		
	T	I	F	T	I	F	T	I	F	T	I	F
T	T	I	F	T	I	T	T	I	F	T	I	F
I	I	I	I	I	I	I	I	I	I	I	I	I
F	F	I	F	T	I	F	T	I	T	F	I	T

First we extend this in the usual way by adopting an assertion operator defined truth-functionally:

p	$\mathrm{A}p$
T	T
I	F
F	F

and using it to define new connectives:

'$\bar{\neg} p$' for '$\neg \mathrm{A}p$'
'$p \mathbin{\hat{\wedge}} q$' for '$\mathrm{A}p \wedge \mathrm{A}q$'
'$p \mathbin{\check{\vee}} q$' for '$\mathrm{A}p \vee \mathrm{A}q$'
'$p \Rightarrow q$' for '$\mathrm{A}p \rightarrow \mathrm{A}q$'
'$p \Leftrightarrow q$' for '$\mathrm{A}p \leftrightarrow \mathrm{A}q$'.

This generates the following matrices:

p	$\bar{\neg} p$
T	F
I	T
F	T

	$p \mathbin{\hat{\wedge}} q$			$p \mathbin{\check{\vee}} q$			$p \Rightarrow q$			$p \Leftrightarrow q$		
	T	I	F	T	I	F	T	I	F	T	I	F
T	T	F	F	T	T	T	T	F	F	T	F	F
I	F	F	F	T	F	F	T	F	T	F	T	T
F	F	F	F	T	F	F	T	T	T	F	T	T

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