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MATRIX CALCULI SS1M AND SS1I COMPARED WITH AXIOMATIC SYSTEMS

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P. Weingartner developed in [1] a modal matrix calculus which he called SS1M; interpreting it in a certain way he obtained the system SS1I. The purpose of the present note is to state the following facts:

1. Propositional (non-modal) SS1I contains intuitionistic propositional logic (but not conversely) and is contained in classical propositional logic (but not conversely).
2. SS1M contains S0.5.
3. SS1M does not contain S0.9 or S1° (and trivially it does not contain any stronger system).
4. S5, K4, and S9 do not contain SS1M.

The reader is supposed to have [1] at hand. Remember that cv means "characteristic value" which is the highest number assigned to a formula by an assignment of elements of $\{1, 2, 3, 4, 5, 6\}$ to the propositional variables of the formula and the value of the composed formula calculated with the help of the various matrices. 1, 2, 3 are designated values. We write also cv(α) to express "the characteristic value of α." By ϕ we mean an assignment of the kind just mentioned and $\phi(\alpha)$ is the respective value of α. The formulations of the various axiom systems are taken from [4] in the case of intuitionistic logic, from [3] in the case of S1°, and from [2] in the other cases.

1 Intuitionistic Propositional Logic is Contained in Propositional SS1I By "propositional SS1I" we mean the "propositional part" of SS1I, that is the set of theorems of SS1I which have as constant symbols only N', A, C', K', and E', but not L or M. Though these connectives are defined in [1], p. 132 with the help of the corresponding classical ones and LM, we can give matrices for them because the definitions are formulated as LL-equivalences (and two LL-equivalent formulas always take the same value for the same assignment ϕ as seen by the matrix for LLE on p. 103 of [1]).

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