Notre Dame Journal of Formal Logic
Volume XV, Number 2, April 1974
NDJFAM

ADMISSIBLE RULES, DERIVABLE RULES, AND EXTENDIBLE LOGISTIC SYSTEMS

HOWARD C. WASSERMAN

*Introduction.** At a 1957 conference at Cornell University of the Summer Institute for Symbolic Logic [3] and in a paper [4] published in 1959, Hiż presented a system of sentential calculus based on the axioms: $\sim(\alpha \supset \beta) \supset \alpha$ and $\sim(\alpha \supset \beta) \supset \sim\beta$, and on the inference rules: $\alpha \supset \beta, \beta \supset \gamma \Rightarrow \alpha \supset \gamma$; $\alpha \supset (\beta \supset \gamma), \alpha \supset \beta \Rightarrow \alpha \supset \gamma$; and $\sim\alpha \supset \beta, \sim\alpha \supset \sim\beta \Rightarrow \alpha$. The system, which shall be herein referred to as **H**, was proven by Hiż to be complete with respect to the usual two-valued matrix $\mathfrak{M}_2$. However, Hiż showed that the system is extendible (i.e., not Post complete); in fact, the system admits infinitely many distinct Post consistent extensions (although, as R. Harrop pointed out, at a meeting in 1958 of the Logic Seminar at Pennsylvania State University, no negation-free formula will extend **H**). What is more, there exist inference rules which are admissible in **H** (i.e., with respect to which the set of theorems of **H** is closed) but which are not derivable in **H** (i.e., it is not the case that every application of such a rule can be *uniformly* replaced by a specific finite application of the primitive rules of **H**). Hiż writes in [4] that "... a result of this paper may be phrased: there is a system of sentential calculus for which if $\alpha_1, \alpha_2, \ldots, \alpha_n \Rightarrow_{mp} \beta$, then $\alpha_1, \alpha_2, \ldots, \alpha_n \Rightarrow_{nmp} \beta$, but not if $\alpha_1, \alpha_2, \ldots, \alpha_n \vdash_{mp} \beta$, then $\alpha_1, \alpha_2, \ldots, \alpha_n \vdash_{nmp} \beta$."[1] Among the admissible non-derivable rules are $\alpha \supset \beta, \alpha \Rightarrow \beta$, $\sim\sim\alpha \Rightarrow \alpha$, $\alpha, \sim\alpha \Rightarrow \beta$, $\alpha, \beta \Rightarrow \alpha \supset \beta$, and $\alpha, \beta \Rightarrow \sim\alpha \supset \sim\beta$.

---

*This work is based on a dissertation in partial fulfillment of the requirements for the Ph.D. degree in Linguistics at the University of Pennsylvania, May 1971. The author wishes to thank Professor Henry Hiż for directing this research.

1. In the terms of the present paper, there is an axiomatic system $A = \langle \mathcal{L}, \mathsf{T}_0, R\rangle$ with *modus ponens* not in R such that any rule admissible in $\mathsf{L}(A')$ is admissible in $\mathsf{L}(A)$, but there is a rule derivable in $\mathsf{L}(A')$ which is not derivable in $\mathsf{L}(A)$, where $A' = \langle \mathcal{L}, \mathsf{T}_0, R\rangle \cup \{\textit{modus ponens}\}$.

Received October 8, 1971