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A CATEGORICAL EQUIVALENCE OF PROOFS

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0 *Introduction.** An intuitionist proof of a sequent $B \rightarrow A$ is essentially a "function," and in this paper we shall study certain properties of the class of such functions. In order to gain sufficient generality, we shall adopt a "multilinear" point of view and take a propositional subsystem of Gentzen's calculus **LJ** as a starting point.

Gentzen's *Hauptsatz* states that for every provable sequent $\Gamma \rightarrow A$, the class $\{P\}$ of **LJ** proofs of $\Gamma \rightarrow A$ contains at least one cut-free representative. We can regard $\{P\}$ as an *equivalence* class with respect to the relation E_0 on proofs in **LJ** defined by $P\mathsf{E}_0Q$ iff P and Q are proofs of the same sequent $\Gamma \rightarrow A$. The question which arises naturally in category theory is to what extent, if at all, E_0 can be refined to an equivalence relation E for a definite propositional fragment of **LJ**, denoted simply by "**L**" below, such that the following are true:

(i) Each E-class has a cut-free representative;
(ii) E separates the structural and operational rules of **L** conservatively;
(iii) If P and Q are two cut-free proofs of the sequent $\Gamma \rightarrow A$, then $P\mathsf{E}Q$ iff P and Q are equi-general, where P and Q are equi-general, roughly speaking, if the terms in the initial sequents of P can be made as distinct as those in Q and conversely without destroying P and Q as *proofs of the same sequent* (but not necessarily of $\Gamma \rightarrow A$).

(i) and (ii) will of course establish immediately certain invariance properties of well-known logical theorems, whereas an *effective* notion of "equi-generality" is needed in order to preserve the decidability of **L**.

Whilst the questions raised in (i), (ii), and (iii) are of logical interest in their own right, the motivation for studying the particular deductive system **L** lies in the fact that **L** constitutes, as is easily deducible from

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