

SCHEMATIZING DE MORGAN'S ARGUMENT

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A century and a quarter ago Augustus De Morgan challenged anyone to deduce syllogistically from 'Every horse is an animal' that 'Every head of a horse is the head of an animal.'¹ His challenge went unanswered, and history gives this argument the credit of being the first to decisively show the shortcomings of Aristotelian logic. Modern logic, encompassing relative terms, can show the formal validity of the argument and is thus rightly thought a great advance. Given the venerability of the argument it is surprising that to the present day it is almost universally schematized incorrectly.

The premise causes no problem, being schematized as

$$(1) \quad (x)(Fx \supset Gx)$$

with appropriate understanding of 'F' and 'G'. It is the conclusion of the argument which is invariably gotten wrong. Look in standard introductory logic texts such as those by Quine, Kalish, and Montague, Copi or Suppes and you will find that this sentence (or its equivalent) is schematized as

$$(2) \quad (y)[(\exists x)(Fx \cdot Hyx) \supset (\exists x)(Gx \cdot Hyx)]$$

where 'Hyx' is read 'y is a head of x'. I think there are reasons showing this is a mistake; further, there are reasons showing why the mistake is not usually noticed.

The correct schematization of the conclusion is

1. I have not found exactly this argument in De Morgan. The argument occurs with 'horse' replaced by 'man', cf. *Formal Logic* (1847), p. 114; *On the Syllogism and Other Logical Writings*, ed. by Peter Heath (Routledge & Kegan Paul, 1966), pp. 29 and 216.

Richard Schubert has pointed out to me that the example with 'horse' occurs in *Principia Mathematica* *37.62 which in turn refers to W. S. Jevons, *Principles of Science* (1887), p. 18. Jevons says De Morgan used the example in conversation.