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COMPLETENESS OF RELEVANT QUANTIFICATION THEORIES

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In [20], Meyer and Dunn answered affirmatively for the relevant sentential logics E and R the question, "Is the rule γ, 'From $\vdash A$ and $\vdash \overline{A} \vee B$, to infer B,' admissible?" This result, which confirmed an old conjecture of Anderson and Belnap, establishes the weak completeness of these and a number of related logics. In the present paper, some of whose principal results were announced without proof in [21], we shall extend the methods of past papers to prove both the admissibility of γ and, in a reasonable sense, weak completeness for the first-order extension RQ of R. In doing so, we replace the intuitively uninformative R-matrices of [20] with the theory of DeMorgan monoids, which furnishes a surprisingly smooth and natural algebraic semantics for R and, by extension, for RQ.

1. Furnishing RQ with a viable algebraic semantics and a proof of γ is no unimportant task. In the first place, the Anderson-Belnap system R of relevant implication is at the sentential level the most stable and interesting of the relevant logics. R contains in exact and well-motivated ways both the intuitionistic and the classical sentential calculi.[1] R_I, the implicational fragment of R,[2] is the oldest of the relevant logics, having been independently investigated twenty years ago by Moh-Shaw-Kwei and by Church in important papers, which provide interesting deductive-methodological motivation (A relevantly implies B only if A is used in some deduction of B).[3]

1. An exact translation of the Curry system HD into R, and hence of the intuitionistic sentential calculus, is presented in [18]; cf. [4] and [17]. In &, v, -, R contains all classical tautologies; cf. [6] and also [1].

2. In unpublished work Meyer has proved that R is a conservative extension of R_I when the latter is axiomatized as by Church in [10]. This settles an open question for R of the sort raised by Anderson for E and E_I in [2]. Cf. also Prawitz's [24].

3. Cf. [10] and [22].

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