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RESOLUTION AND THE CONSISTENCY OF ANALYSIS

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§1. *Introduction.** In [2] we formulated a system $\mathcal{R}$, called a Resolution system, for refuting finite sets of sentences of type theory, and proved that $\mathcal{R}$ is complete in the (weak) sense that every set of sentences which can be refuted in the system $\mathcal{T}$ of type theory due to Church [5] can also be refuted in $\mathcal{R}$. The statement that $\mathcal{R}$ is in this sense complete is a purely syntactic one concerning finite sequences of wffs. However, it is clear that there can be no purely syntactic proof of the completeness of $\mathcal{R}$, since the completeness of $\mathcal{R}$ is closely related to Takeuti's conjecture [9] (since proved by Takahashi [8] and Pravitz [7]) concerning cut-elimination in type theory. As Takeuti pointed out in [9] and [10], cut-elimination in type theory implies the consistency of analysis. Indeed, Takeuti's conjecture implies the consistency of a formulation of type theory with an axiom of infinity; in such a system classical analysis and much more can be formalized. Hence, to avoid a conflict with Gödel's theorem, any proof of the completeness of resolution in type theory must involve arguments which cannot be formalized in type theory with an axiom of infinity. Indeed, the proof in [2] does involve a semantic argument. Nevertheless, it must be admitted that anyone who does not find the line of reasoning sketched above completely clear will have difficulty finding a unified and coherent exposition of the entire argument in the published literature. We propose to remedy this situation here.

We presuppose familiarity with §2 (The System $\mathcal{T}$) and Definitions 4.1 and 5.1 (The Resolution System $\mathcal{R}$) of [2], and follow the notation used there. In particular, $\Box$ stands for the contradictory sentence $\forall p_o p_o$. To distinguish between formulations of $\mathcal{T}$ with different sets of parameters, we henceforth assume $\mathcal{T}$ has no parameters, and denote by $\mathcal{T}(\mathbf{A}^1, \ldots, \mathbf{A}^n)$ a formulation of the system with parameters $\mathbf{A}^1, \ldots, \mathbf{A}^n$. If $\mathcal{H}$ is a set of sentences, $\mathcal{H} \vdash_{\mathcal{S}} \mathbf{B}$ shall mean that $\mathbf{B}$ is derivable from some finite subset of $\mathcal{H}$ in system $\mathcal{S}$. The deduction theorem is proved in §5 of [5]. We shall

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