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KRIPKE'S DEONTIC SEMANTICS AGAIN

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Routley and Montgomery [5] have noted that Kripke's purported model structures for Lemmon's D2 and D3 [1], p. 220, are inadequate. Instead, they say, Kripke's D2 structures characterize E2. Actually, however, those structures model Lemmon's C2 [2], as is evident from a careful comparison of Kripke's text with Routley's own [4].

Routley and Montgomery: "Kripke's model structure for D2 is a quadruple $\langle G, K, R, N\rangle$ where N is the set of normal elements, i.e., the subset of K such that for every H in N, HRH" [5].

The authors' mistake here is to have used Kripke's first, more restrictive definition of normality (as self-accessibility), whereas in his remarks on D2 [1], p. 220, Kripke has reference to his second, more general, parenthesized definition of $\mathbf{N}$ simply as a subset of $\mathbf{K}$ [1], p. 211, as his forward reference *ibid.* to §7 indicates. For E2 Kripke then stipulates further that $\boldsymbol{R}$ be *reflexive on* $\mathbf{N}$. For D2 the requirement is dropped: "if we drop quasi-reflexivity from the requirements on $\boldsymbol{R}$, and simply define a model structure to be a quadruple $(\mathbf{G}, \mathbf{K}, \boldsymbol{R}, \mathbf{N})$, where $\mathbf{N}$ is the set of normal elements, we get a model theory for Lemmon's D2" [1], p. 220.

Kripke has inadvertently dropped the *on* with the *reflexive*. He fails to retain the requirement that $\mathbf{N} \subseteq \mathrm{D}`\boldsymbol{R}$: every normal world must have an alternative (Routley's condition R2 [4], p. 241). The models resulting from this oversight have been shown by Routley [4], p. 243, to characterize C2, i.e., Lemmon's D2 minus the axiom (2′) $\Box A \supset \Diamond A$.

Kripke continues, "If $\boldsymbol{R}$ is required to be transitive, the theory works for D3." This is not so even granting condition R2. Lemmon's D3 axiom (D), $\Box M \supset M$ for fully modalized M [3], p. 184, remains invalid; *cf.* the instance $\Box\Box A \supset \Box A$. Again, Routley's [4] shows that what Kripke's D3 model structures really characterize is C3, as we might call C2 plus the distinctive S3-axiom. If R2 is imposed, they model D2 as likewise augmented, i.e., Lemmon's D3 minus (D).

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