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A NEW EXTENSION OF S4

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In this paper it is shown that the addition to S4 of the axiom

Γ1 $MLp \rightarrow (LMp \rightarrow LMLp)$

generates a new system, to be called S4.01, that is contained in every known extension of S4 except S4.02 and S4.04. To prove this it suffices to derive **Γ1** in S4.1, S4.2, and Z1, since all S4-extensions other than S4.02 and S4.04 contain at least one of these three systems.

In the field of S4 there are a number of interesting formulae that are deductively equivalent to **Γ1**. These include

Γ2 $MLp \rightarrow L(LMp \rightarrow MLp)$
Γ3 $ML(p \rightarrow Lp) \rightarrow L(LMp \rightarrow MLp)$
Γ4 $(LMp \rightarrow MLp) \rightarrow L(LMp \rightarrow MLp)$

Proof:

(1)	$L(LMp \rightarrow MLp) \rightarrow (LMp \rightarrow LMLp)$	S4
(2)	$LML(p \rightarrow Lp) \rightarrow L(LMp \rightarrow MLp)$	S4
(3)	$LM(p \rightarrow Lp)$	S2
Γ2	$MLp \rightarrow L(LMp \rightarrow MLp)$	**Γ4**, PC
Γ1	$MLp \rightarrow (LMp \rightarrow LMLp)$	**Γ2**, (1)
(4)	$ML(p \rightarrow Lp) \rightarrow (LM(p \rightarrow Lp) \rightarrow LML(p \rightarrow Lp))$	**Γ1**, $p/p \rightarrow Lp$
Γ3	$ML(p \rightarrow Lp) \rightarrow L(LMp \rightarrow MLp)$	(2), (3), (4)
(5)	$(L \sim p \vee Lp) \rightarrow L(\sim p \vee Lp)$	S4
(6)	$M(Mp \rightarrow Lp) \rightarrow ML(p \rightarrow Lp)$	(5), C2
Γ4	$(LMp \rightarrow MLp) \rightarrow L(LMp \rightarrow MLp)$	**Γ3**, (6), C2

The substitution $p/\sim p$ in **Γ2**, and simple transformations show that yet another axiom for S4.01 is

Γ5 $LMp \vee L(LMp \rightarrow MLp)$

Γ1 is easily derivable from the S4.2 axiom

G2 $MLp \rightarrow LMLp$

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