Notre Dame Journal of Formal Logic
Volume XIV, Number 4, October 1973
NDJFAM

A NOTE ON "TRANSITIVITY, SUPERTRASITIVITY AND INDUCTION"

W. RUSSELL BELDING and RICHARD L. POSS

In the review of our paper "Transitivity, Supertransitivity and Induction," [1] that occurred in [2], the reviewer pointed out two apparent errors. We will here clarify the points in mention.

The reviewer first stated that Lemma 9 "seems to be in error." The difficulty, as we see it, is that the transition from step (1) to step (2) was unclear, so we will present a somewhat more complete proof. We will assume

(1) $(y)(y \in \mathsf{Fld}\,\epsilon_S \wedge (x)(x \epsilon\ y \rightarrow \varphi(x)) \rightarrow \varphi(y)) \rightarrow (y)(y \in \mathsf{Fld}\,\epsilon_S \rightarrow \varphi(y))$

for formulas $\varphi(x)$ not containing y or u and show that

(2) $(u)(u \in \mathsf{Fld}\,R \wedge (v)(vRu \rightarrow \varphi(v)) \rightarrow \varphi(u)) \rightarrow (u)(u \in \mathsf{Fld}\,R \rightarrow \varphi(u))$

for formulas $\varphi(x)$ not containing y or u. This would conclude the proof of the lemma. We now suppose the hypothesis of (2); i.e., we assume that

(3) $(u)(u \in \mathsf{Fld}\,R \wedge (v)(vRu \rightarrow \varphi(v)) \rightarrow \varphi(u))$

where $\varphi(v)$ does not contain y or u. It remains to show that

(4) $(u)(u \in \mathsf{Fld}\,R \rightarrow \varphi(u))$.

We now define the formula ψ as follows:

(5) $\psi(x) \equiv x \in \mathsf{Fld}\,\epsilon_S \wedge \varphi(f'x)$.

We will first show that ψ satisfies the hypothesis of (1). Suppose that

(6) $y \in \mathsf{Fld}\,\epsilon_S$

and

(7) $(x)(x \in y \rightarrow \psi(x))$.

We must show that $\psi(y)$. It is clear from (6) that the first part of the definition of ψ is satisfied. It remains to show that $\varphi(f'y)$. Since f is an isomorphism, there exists u such that

Received April 24, 1973